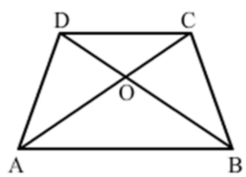
Chapter-Triangles
Question bank

Q1.

D and E are points on the sides AB and AC respectively of a ΔABC such that DE ∥ BC. Find the value of x when AD = x cm, DB = (x − 2) cm, AE = (x + 2) cm and EC = (x − 1) cm.

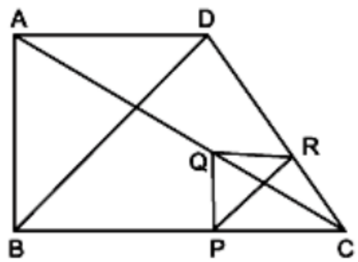
Q2.

In the adjoining figure, ABCD is a trapezium in which CD ∥ AB and its diagonals intersect at O. If AO = (2x + 1) cm, OC = (5x – 7) cm, DO = (7x – 5) cm and OB = (7x + 1) cm, find the value of x.



Q3.

In figure, two triangles ABC and DBC lie on the same side of base BC. P is a point on BC such that PQ || BA and PR || BD. Prove that QR || AD.



Q4.

P and Q are points on the sides AB and AC respectively of a triangle ABC. If AP = 2 cm, PB = 4 cm, AQ = 3cm, QC = 6 cm, prove that BC = 3PQ.

Q5.

A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Q6.

State and prove Basic proportionality Theorem.

Q7.

ABC is a right-angled triangle, right-angled at A. A circle is inscribed in it. The lengths of the two sides containing the right angle are 6cm and 8 cm. Find the radius of the in circle.

Q8.

X and Y are points on the sides AB and AC, respectively of a triangle ABC such that $\frac{AX}{AB}$, $AY = 2$ cm and $YC = 6$ cm. Find whether $XY \parallel BC$ or not.

Q9.

In the given figure, ∠AEF=∠AFE and E is the mid-point of CA. Prove that

$$\frac{BD}{CD} = \frac{BF}{CE}$$

Q10.

Let ABC be a triangle D and E be two points on side AB such that $AD = BE$. If $DP \parallel BC$ and $EQ \parallel AC$, then prove that $PQ \parallel AB$.

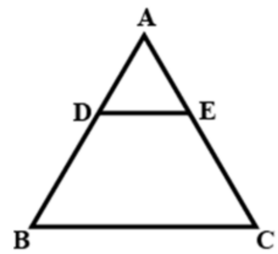
[Board Term-1 2012]

Solutions

Solution 1.

In △ABC, it is given that DE ∥ BC.

By Basic Proportionality Theorem, we have $\frac{AD}{DB} = \frac{AE}{EC}$



$$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$$

$\Rightarrow x(x-1) = (x-2)(x+2)$

$\Rightarrow x^2 - x = x^2 - 4$

$\Rightarrow x = 4$ cm

Solution 2.

△AOB ~ △COD (By AA Similarity as ∠OAB = ∠OCD and ∠OBA = ∠ODC – Alternate Interior angles)

So, their corresponding sides are proportional

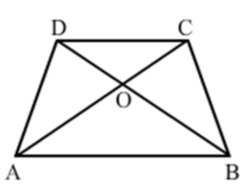
$\therefore \frac{AO}{OC} = \frac{BO}{OD} \Rightarrow \frac{2x+1}{5x-7} = \frac{7x+1}{7x-5} \Rightarrow (5x-7)(7x+1) = (7x-5)(2x+1)$

$\Rightarrow 35x^2 + 5x - 49x - 7 = 14x^2 - 10x + 7x - 5 \Rightarrow 21x^2 - 41x - 2 = 0$

$\Rightarrow 21x^2 - 42x + x - 2 = 0 \Rightarrow 21x(x-2) + 1(x-2) = 0$

$\Rightarrow (x-2)(21x+1) = 0 \Rightarrow x = 2, -\frac{1}{21}$

$\because x \neq -\frac{1}{21} \quad \therefore x = 2$



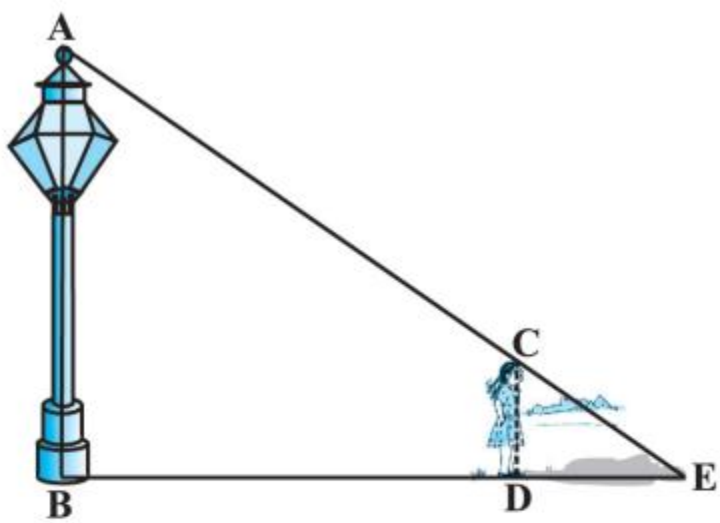
Solution 3:

Ans:
Given : In ΔABC, PQ || AB and PR || BD
To prove : QR || AD
Proof : By BPT $\frac{CP}{BP}=\frac{CQ}{AQ}$ ...(i)
Now in ΔBCD, PR || BD
$\Rightarrow$ By using BPT $\frac{CP}{BP}=\frac{CR}{RD}$ ...(ii)
From (i) and (ii), $\frac{CQ}{AQ}=\frac{CR}{RD} \Rightarrow$ By converse of BPT, QR || AD

Solution 4.

Ans: $\frac{AP}{PB}=\frac{2}{4}=\frac{1}{2}$ ...(i) $\frac{AQ}{QC}=\frac{3}{6}=\frac{1}{2}$ ...(ii)
From (i) and (ii) $\frac{AP}{PB}=\frac{AQ}{QC} \Rightarrow PQ \parallel BC$
In ΔABC and ΔAPQ.
$\frac{AB}{AP}=\frac{AC}{AQ}$ ($\because$ PQ || BC)
$\angle A=\angle A$ (Common)
$\therefore$ $\Delta ABC \sim \Delta APQ$, (SAS similarity)
$\Rightarrow \frac{AB}{AP}=\frac{BC}{PQ} \Rightarrow \frac{AP+PB}{AP}=\frac{BC}{PQ}$
$\Rightarrow \frac{2+4}{2}=\frac{BC}{PQ} \Rightarrow \frac{6}{2}=\frac{BC}{PQ}=\frac{1}{3} \Rightarrow BC=3PQ$. Hence proved.

Solution 5.



Let AB denote the lamp-post and CD the girl after walking for 4 seconds away from the lamp-post. From the figure, you can see that DE is the shadow of the girl. Let DE be x metres.

Now, BD = 1.2 m × 4 = 4.8 m.

Note that in Δ ABE and Δ CDE,

$\angle B = \angle D$ (Each is of 90° because lamp-post as well as the girl are standing vertical to the ground)

and $\angle E = \angle E$ (Same angle)

So, $\Delta ABE \sim \Delta CDE$ (AA similarity criterion)

Therefore, $\frac{BE}{DE} = \frac{AB}{CD}$

i.e., $\frac{4.8 + x}{x} = \frac{3.6}{0.9}$ $\left(90 \text{ cm} = \frac{90}{100} \text{ m} = 0.9 \text{ m}\right)$

i.e., $4.8 + x = 4x$

i.e., $3x = 4.8$

i.e., $x = 1.6$

So, the shadow of the girl after walking for 4 seconds is 1.6 m long.

Solution 6.

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Proof : We are given a triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively (see Fig. 6.10).

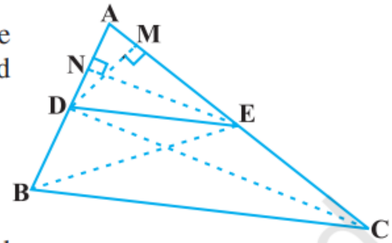


Fig. 6.10

We need to prove that $\frac{AD}{DB} = \frac{AE}{EC}$.

Let us join BE and CD and then draw DM ⊥ AC and EN ⊥ AB.

Now, area of Δ ADE (= $\frac{1}{2}$ base × height) = $\frac{1}{2}$ AD × EN.

Recall from Class IX, that area of Δ ADE is denoted as ar(ADE).

So, $$\text{ar(ADE)} = \frac{1}{2}\,\text{AD} \times \text{EN}$$

Similarly, $$\text{ar(BDE)} = \frac{1}{2}\,\text{DB} \times \text{EN},$$

$$\text{ar(ADE)} = \frac{1}{2}\,\text{AE} \times \text{DM} \quad \text{and} \quad \text{ar(DEC)} = \frac{1}{2}\,\text{EC} \times \text{DM}.$$

Therefore, $$\frac{\text{ar(ADE)}}{\text{ar(BDE)}} = \frac{\frac{1}{2}\,\text{AD} \times \text{EN}}{\frac{1}{2}\,\text{DB} \times \text{EN}} = \frac{\text{AD}}{\text{DB}} \qquad (1)$$

and $$\frac{\text{ar(ADE)}}{\text{ar(DEC)}} = \frac{\frac{1}{2}\,\text{AE} \times \text{DM}}{\frac{1}{2}\,\text{EC} \times \text{DM}} = \frac{\text{AE}}{\text{EC}} \qquad (2)$$

Note that Δ BDE and DEC are on the same base DE and between the same parallels BC and DE.

So, $$\text{ar(BDE)} = \text{ar(DEC)} \qquad (3)$$

Therefore, from (1), (2) and (3), we have :

$$\frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$$

Solution 7.

(Ans: r=2)

BC = 10cm
y + z = 8cm
x + z = 6cm
x + y = 10
$\Rightarrow x + y + z = 12$
z = 12 – 10
z = 2 cm
$\therefore$ radius = 2cm

Solution 8.

$$\frac{AX}{AB} = \frac{1}{4}$$

i.e., $AX = 1K, \text{AB} = 4K$

(K– constant)

$\therefore$ $BX = AB - AX$

$= 4K - 1K = 3K$

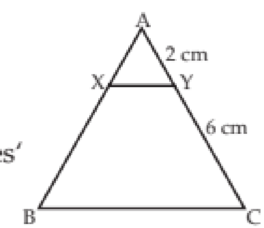
Now, $\frac{AX}{XB} = \frac{1K}{3K} = \frac{1}{3}$

and, $\frac{AY}{YC} = \frac{2}{6} = \frac{1}{3}$

$$\frac{AX}{XB} = \frac{AY}{YC}$$

$\therefore$ $XY \parallel BC$

(By converse of Thales' theorem)



Solution 9.

ns: Draw CG ∥ DF

In Δ BDF

CG ∥ DF

$\therefore \frac{BD}{CD}=\frac{BF}{GF}$ ....(1) BPT

In ΔAFE

∠AEF=∠AFE

⇒AF=AE

⇒AF=AE=CE..............(2)

In Δ ACG

E is the mid point of AC

⇒ FG = AF

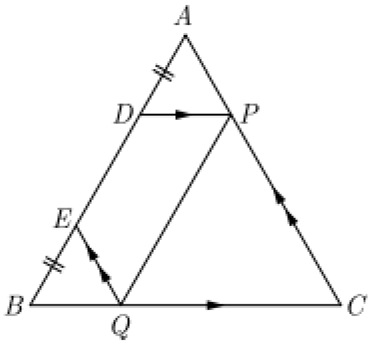
∴ From (1) & (2)

$$\frac{BD}{CD} = \frac{BF}{CE}$$

Hence proved

Solution 10.

As per given condition we have drawn the figure below.



In ΔABC, $DP \parallel BC$

By BPT we have $\frac{AD}{DB} = \frac{AP}{PC}$, ...(1)

Similarly, in ΔABC, $EQ \parallel AC$

$$\frac{BQ}{QC} = \frac{BE}{EA} \quad ...(2)$$

From figure, $EA = AD + DE$

$= BE + ED$ $(\because BE = AD)$

$= BD$

Therefore equation (2) becomes,

$$\frac{BQ}{QC} = \frac{AD}{BD} \quad ...(3)$$

From (1) and (3), we have

$$\frac{AP}{PC} = \frac{BQ}{QC}$$

By converse of BPT,

$PQ \parallel AB$ Hence Proved