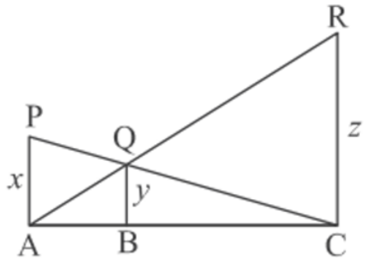
Chapter-Triangles
Question bank

Q1. In the given figure PA, QB and RC are each perpendicular to AC. If AP = x, BQ = y and CR = z, then

prove that $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$



Q2.

ABC is an isosceles triangle in which $AB = AC = 10$ cm $BC = 12$ cm $PQRS$ is a rectangle inside the isosceles triangle. Given $PQ = SR = y$, $PS = PR = 2x$. Prove that $x = 6 - \frac{3y}{4}$.

Q3.

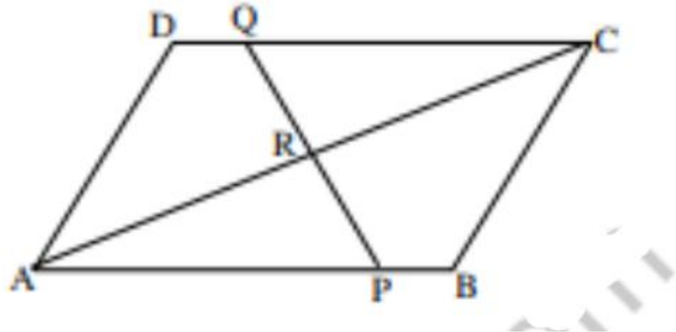
If A be the area of a right triangle and b be one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\frac{2Ab}{\sqrt{b^4 + 4A^2}}$.

Q4.

In a trapezium $ABCD$, $AB || DC$ and $DC = 2AB$. $EF = AB$, where E and F lies on BC and AD respectively such that $\frac{BE}{EC} = \frac{4}{3}$ diagonal DB intersects EF at G. Prove that, $7EF = 11AB$.

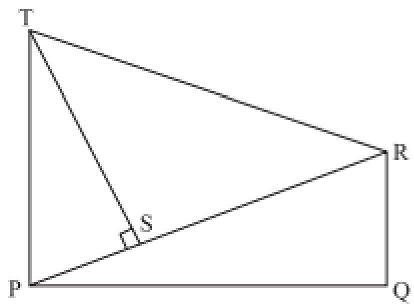
Q5.

ABCD is a parallelogram in the given figure, AB is divided at P and CD and Q so that AP:PB=3:2 and CQ:QD=4:1. If PQ meets AC at R, prove that AR=$\frac{3}{7}$AC.



Q6.

. In the given figure, RQ and TP are perpendicular to PQ, also $TS \perp PR$ prove that $ST \cdot RQ = PS \cdot PQ$.



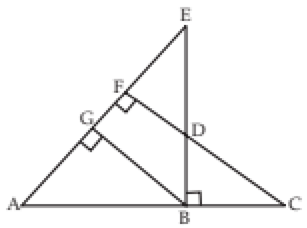
Q7.

. In given figure, $EB \perp AC$, $BG \perp AE$ and $CF \perp AE$.

Prove that:

(i) $\Delta ABG \sim \Delta DCB$

(ii) $\frac{BC}{BD} = \frac{BE}{BA}$

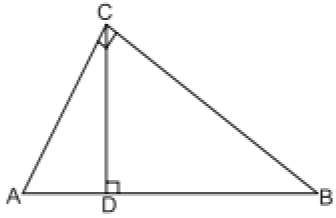


Q8.

Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that $AP \times PC = BP \times DP$.

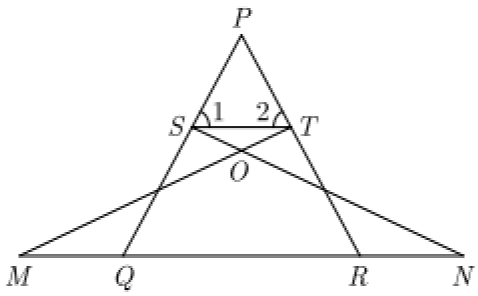
Q9.

. **In Fig. $\angle ACB = 90°$ and $CD \perp AB$, prove that $CD^2 = BD \times AD$.**



Q10.

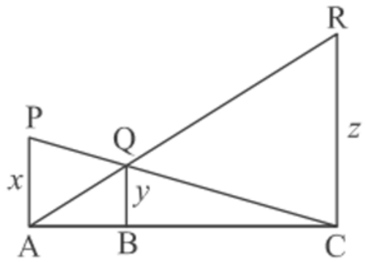
.In given figure $\angle 1 = \angle 2$ and $\Delta NSQ - \Delta MTR$, then prove that $\Delta PTS - \Delta PRO$.



SOLUTIONS

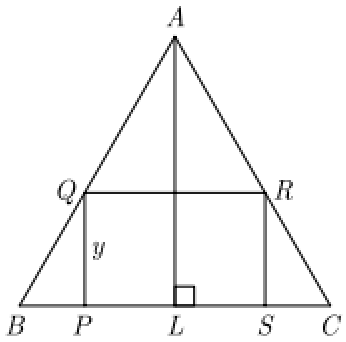
Q1. In the given figure PA, QB and RC are each perpendicular to AC. If AP = x, BQ = y and CR = z, then

prove that $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$



Q2.

As per given condition we have drawn the figure below.



Here we have drawn $AL \perp BC$.
Since it is isosceles triangle, AL is median of BC,

$$BL = LC = 6 \text{ cm}.$$

In right ΔALB, by Pythagoras theorem,

$$AL^2 = AB^2 - BL^2$$

$$= 10^2 - 6^2 = 64 = 8^2$$

Thus $AL = 8$ cm.

In ΔBPQ and ΔBLA, angle $\angle B$ is common and

$$\angle BPQ = \angle BLA = 90^\circ$$

Thus by AA similarity we get

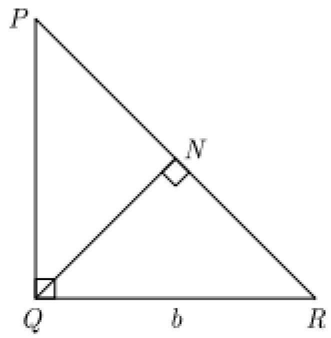
$$\Delta BPQ \sim \angle BLA$$

$$\frac{PB}{PQ} = \frac{BL}{AL}$$

$$\frac{6-x}{y} = \frac{6}{8}$$

$x = 6 - \frac{3y}{4}$ Hence proved.

Q3.



Let $QR = b$, then we have

$$A = ar(\Delta PQR)$$
$$= \frac{1}{2} \times b \times PQ$$
$$PQ = \frac{2 \cdot A}{b} \qquad ...(1)$$

Due to AA similarity we have

$$\Delta PNQ \sim \Delta PQR$$
$$\frac{PQ}{PR} = \frac{NQ}{QR} \qquad ...(2)$$

From ΔPQR

$$PQ^2 + QR^2 = PR^2$$
$$\frac{4A^2}{b^2} + b^2 = PR^2$$
$$PR = \sqrt{\frac{4A^2 + b^4}{b^2}}$$

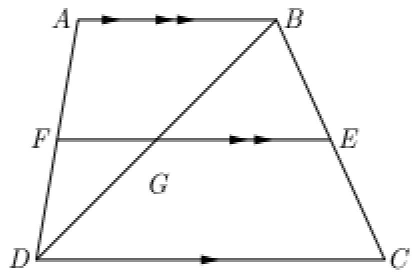
Equation (2) becomes

$$\frac{2A}{b \times PR} = \frac{NQ}{b}$$
$$NQ = \frac{2A}{PR}$$

Altitude, $NQ = \frac{2Ab}{\sqrt{4A^2 + b^4}}$ Hence Proved.

Q4.

As per given condition we have drawn the figure below.



In trapezium $ABCD$,

$$AB \parallel DC \text{ and } DC = 2AB.$$

Also, $$\frac{BE}{EC} = \frac{4}{3}$$

Thus $$EF \parallel AB \parallel CD$$

$$\frac{AF}{FD} = \frac{BE}{EC} = \frac{4}{3}$$

In ΔBGE and ΔBDC, $\angle B$ is common and due to corresponding angles,

$$\angle BEG = \angle BCD$$

Due to AA similarity we get

$$\Delta BGE \sim \Delta BDC$$

, $$\frac{EG}{CD} = \frac{BE}{BC} \qquad ...(1)$$

As, $$\frac{BE}{EC} = \frac{4}{3}$$

$$\frac{BE}{BE+EC} = \frac{4}{4+3} = \frac{4}{7}$$

$$\frac{BE}{BC} = \frac{4}{7} \qquad ...(2)$$

From (1) and (2) we have

$$\frac{EG}{CD} = \frac{4}{7}$$

, $$EG = \frac{4}{7}CD \qquad ...(3)$$

Similarly, $\Delta DGF \sim \Delta DBA$

$$\frac{DF}{DA} = \frac{FG}{AB}$$

$$\frac{FG}{AB} = \frac{3}{7}$$

$$FG = \frac{3}{7}AB \qquad ...(4)$$

$$\left[\frac{AF}{AD} = \frac{4}{7} = \frac{BE}{BC} \Rightarrow \frac{EC}{BC} = \frac{3}{7} = \frac{DE}{DA}\right]$$

Adding equation (3) and (4) we have

$$EG + FG = \frac{4}{7}DC + \frac{3}{7}AB$$

$$EF = \frac{4}{7} \times (2AB) + \frac{3}{7}AB$$

$$= \frac{8}{7}AB + \frac{3}{7}AB = \frac{11}{7}AB$$

$$7EF = 11AB$$ Hence proved.

Q5.

Ans: $\Delta APR \sim \Delta CQR$ (AA)

$$\Rightarrow \frac{AP}{CQ}=\frac{PR}{QR}=\frac{AR}{CR}$$

$$\Rightarrow \frac{AP}{CQ}=\frac{AR}{CR} \ \& \ AP=\frac{3}{5}AB$$

$$\Rightarrow \frac{3AB}{5CQ}=\frac{AR}{CR} \ \& \ CQ=\frac{4}{5}CD=\frac{4}{5}AB$$

$$\Rightarrow \frac{AR}{CR}=\frac{3}{4}$$

$$\Rightarrow \frac{CR}{AR}=\frac{4}{3}$$

$$\frac{CR+AR}{AR}=\frac{4}{3}+1$$

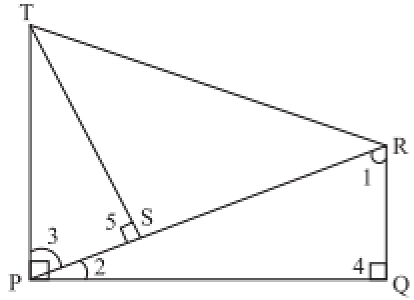
$$\Rightarrow \frac{AC}{AR}=\frac{7}{3}$$

$$\Rightarrow AR=\frac{3}{7}AC$$

Hence proved

Q6.

Ans.



In ΔRPQ,

$$\angle 1 + \angle 2 + \angle 4 = 180^\circ$$

$$\Rightarrow \angle 1 + \angle 2 + 90^\circ = 180^\circ$$

$$\Rightarrow \angle 1 + \angle 2 = 180^\circ - 90^\circ$$

$$\Rightarrow \angle 1 = 90^\circ - \angle 2 \quad \text{...(i)}$$

$$\because TP \perp PQ$$

$$\therefore \angle TPQ = 90^\circ$$

$$\Rightarrow \angle 2 + \angle 3 = 90^\circ$$

$$\Rightarrow \angle 3 = 90^\circ - \angle 2 \quad \text{...(ii)}$$

From eq. (i) and eq. (ii),

$$\angle 1 = \angle 3$$

Now in ΔRQP and ΔPST,

$$\angle 1 = \angle 3 \quad \text{[Proved above]}$$

$$\angle 4 = \angle 5 \quad \text{[Each } 90^\circ]$$

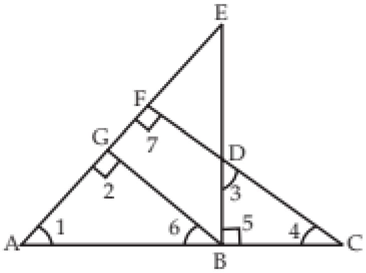
So by AA similarity

$$\Delta RQP \sim \Delta PST$$

$$\frac{ST}{QP} = \frac{PS}{RQ}. \quad \text{[By c.p.c.t.]}$$

$$\Rightarrow ST \cdot RQ = PS \cdot PQ$$ **Hence Proved.**

Q7.



Given: $EB \perp AC$, $BG \perp AE$ and $CF \perp AE$

To prove: (i) $\Delta ABG \sim \Delta DCB$

(ii) $$\frac{BC}{BD} = \frac{BE}{BA}$$

Proof: (i) In ΔABG and ΔDCB, $BG \parallel CF$ as corresponding angles 2 and 7 are equal.

$\angle 2 = \angle 5$ [Each 90°]

$\angle 6 = \angle 4$

[Corresponding angles]

$\therefore$ $\Delta ABG \sim \Delta DCB$ Hence Proved.

[By AA similarity]

$\therefore$ $\angle 1 = \angle 3$ [c.p.c.t]

(ii) In ΔABE and ΔDBC,

$\angle 1 = \angle 3$ [Proved above]

$\angle ABE = \angle 5$

[Each is 90°, $EB \perp AC$ (Given)]

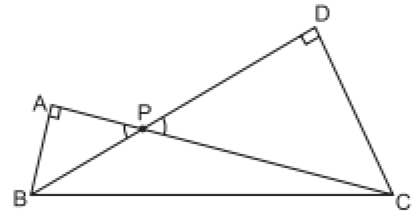
$\therefore$ $\Delta ABE \sim \Delta DBC$ [By AA similarity]

In similar triangles, corresponding sides are proportional

$\therefore$ $\frac{BC}{BD} = \frac{BE}{BA}$ **Hence Proved.**

Q8.

Given, ΔABC and ΔDBC are right angle triangles, right angled at A and D respectively, on same side of BC. AC & BD intersect at P.



In ΔAPB and ΔPDC,

$$\angle A = \angle D = 90°$$

$$\angle APB = \angle DPC \quad \text{(Vertically opposite)}$$

$$\therefore \quad \Delta APB \sim \Delta DPC \quad \text{(By AA Similarity)}$$

$$\therefore \quad \frac{AP}{BP} = \frac{PD}{PC} \quad \text{(by c.s.s.t.)}$$

$\Rightarrow AP \times PC = BP \times PD.$ **Hence Proved.**

Q9.

Given, ΔACB in which $\angle ACB = 90°$ and $CD \perp AB$

To prove: $CD^2 = BD \times AD$

Proof: In ΔADC and ΔACB

$$\angle A = \angle A \quad \text{(common)}$$

$$\angle ADC = \angle ACB \quad \text{(90° each)}$$

$$\therefore \quad \Delta ADC \sim \Delta ACB \quad \text{(By AA rule)}$$

$$\Rightarrow \quad \frac{AD}{CD} = \frac{AC}{BC} \quad \text{...(i)}$$

Similarly,

$$\Delta CDB \sim \Delta ACB \quad \text{(By AA rule)}$$

$$\Rightarrow \quad \frac{AD}{CD} = \frac{AC}{BC} \quad \text{...(ii)}$$

From equation (i) and (ii)

$$\frac{AD}{CD} = \frac{CD}{DB}$$

$$\Rightarrow \quad CD^2 = AD \cdot BD$$

$\Rightarrow \quad CD^2 = BD \times AD$ **Hence Proved.**

Q10.

We have $\Delta NSQ \cong \Delta MTR$

By CPCT we have

$$\angle SQN = \angle TRM$$

From angle sum property we get

$$\angle P + \angle 1 + \angle 2 = \angle P + \angle PQR + \angle PRQ$$

$$\angle 1 + \angle 2 = \angle PQR + \angle PRQ$$

Since $\angle 1 = \angle 2$ and $\angle PQR = \angle PRQ$ we get

$$2\angle 1 = 2\angle PQR$$

$$\angle 1 = \angle PQR$$

Also $\angle 2 = \angle QPR$ (common)

Thus by AAA similarity,

$$\Delta PTS \sim \Delta PRQ$$