Chapter-Surface area and volume

Surface Areas and Volumes - Formulas

Cube

- Side length: a
- Lateral surface area: $4a^2$
- Total Surface Area: $6a^2$
- Volume: a^3

Cuboid

- Length: l, Width: b, Height: h
- Lateral surface area: $2h(l+b)$
- Total Surface Area: $2(lb + lh + bh)$
- Volume: $l \times b \times h$

Sphere

- Radius: r
- Surface Area: $4\pi r^2$
- Volume: $(4/3)\pi r^3$

Hemisphere

- Radius: r
- Surface Area (including base): $3\pi r^2$
- Curved Surface Area (excluding base): $2\pi r^2$
- Volume: $(2/3)\pi r^3$

Cylinder

- Radius: r, Height: h
- Total Surface Area: $2\pi r^2 + 2\pi rh$
- Curved Surface Area: $2\pi rh$
- Volume: πr^2h

Cone

- Radius: r, Height: h, Slant height: l (where $l^2 = r^2 + h^2$)
- Total Surface Area: $\pi r^2 + \pi rl$
- Curved Surface Area: πrl
- Volume: $(1/3)\pi r^2 h$

Combinations of Solids

When dealing with combinations of solids, the key is to break down the complex shape into its constituent parts and apply the appropriate formulas for each part. Here are some common approaches:

1. Addition Method: Add the volumes or surface areas of individual components.
   Example: For a cylinder with hemispheres on both ends:
   - Total Volume = Volume of cylinder + Volume of two hemispheres
   - Surface Area = Curved surface area of cylinder + Surface area of two hemispheres
2. Subtraction Method: Subtract the volume or surface area of one shape from another.
   Example: For a hemisphere with a cylindrical hole:
   - Total Volume = Volume of hemisphere - Volume of cylinder
   - Surface Area = Surface area of hemisphere + Area of circular base + Curved surface area of cylinder
3. Composite Shapes: Identify overlapping regions to avoid double-counting.
   Example: For a cone placed on top of a cylinder:
   - Total Volume = Volume of cone + Volume of cylinder
   - Surface Area = Curved surface area of cone + Curved surface area of cylinder + Area of base of cylinder (Note: The base of the cone is not included as it's shared with the top of the cylinder)
4. Hollow Shapes: Calculate the difference between the outer and inner volumes or surface areas.
   Example: For a hollow sphere:
   - Volume = Volume of outer sphere - Volume of inner sphere
   - Surface Area = Surface area of outer sphere + Surface area of inner sphere

Example 1: A toy is in the shape of a cylinder with two hemispheres stuck to each of its circular faces. The height of the cylinder is 10 cm and the diameter of the circular face is 7 cm. Find the surface area of the toy.

Solution:

Radius (r) = 7/2 = 3.5 cm

Cylinder's curved surface area = $2\pi rh = 2\pi(3.5)(10) = 70\pi\ cm^2$

Area of two hemispheres = $2 \times 2\pi r^2 = 4\pi(3.5^2) = 49\pi\ cm^2$

Total surface area = $70\pi + 49\pi = 119\pi \approx 373.85\ cm^2$

Example 2: A container is in the form of a cylinder surmounted by a hemisphere. The total height of the container is 25 cm and the diameter of the cylinder is 14 cm. Find the total surface area of the container.

Solution:

Radius (r) = 14/2 = 7 cm

Height of cylinder (h) = 25 - 7 = 18 cm (total height minus hemisphere's height)

Curved surface area of cylinder = $2\pi rh = 2\pi(7)(18) = 252\pi\ cm^2$

Area of hemisphere = $2\pi r^2 = 2\pi(7^2) = 98\pi\ cm^2$

Area of circular base = $\pi r^2 = \pi(7^2) = 49\pi\ cm^2$

Total surface area = $252\pi + 98\pi + 49\pi = 399\pi \approx 1253.23\ cm^2$