Chapter-Surface area and volume

Q1.
A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part.

Q2.
A sphere of maximum volume is cut out from a solid hemisphere of radius r. What will be the ratio of the volume of hemisphere to that of the sphere?

Q3.
The height of the cylinder in the figure is h. Two cones are formed as shown with heights **3h/4** *and* **h/4**. *Write the ratio of the volume of the bigger cone to the smaller cone.*

Q4.
A solid is in the shape of a cone surmounted on a hemisphere. The radius of each of them being 3.5 cm and the total height of the solid is 9.5 cm. Find the volume of the solid

Q5.
A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 20 cm and the diameter of the cylinder is 7 cm. Find the total volume of the solid. (Use $\pi = 22/7$).

Q6.
A sphere of diameter 6 cm is dropped into a cylindrical vessel, partly filled with water, whose diameter is 12 cm. If the sphere is completely submerged in water, by how much will the surface of water be raised in the cylindrical vessel?

Q7.
A solid sphere of radius 10.5 cm is melted and recast into smaller solid cones, each of radius 3.5 cm and height 3 cm. Find the number of cones so formed.

Q8.
A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal,then find the ratio of the radius and the height of the conical part

Q9.

A hemispherical depression is cut out from one face of a cubical block of side 7 cm, such that the diameter of the hemisphere is equal to the edge of the cube. Find the surface area of remaining solid

Q10.

From a solid cylinder of height 2.8 cm and diameter 4.2 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid

Solutions

Q1.

.The ratio of the volumes of two spheres is 8 : 27. If r and R are the radii of sphere respectively, then find the $(R-r):r$.

Ans :

Ratio of volumes

$$\frac{\text{Volume of } 1^{st}\text{sphere}}{\text{Volume of } 2^{nd}\text{sphere}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{8}{27}$$

$$\frac{r^3}{R^3} = \frac{8}{27}$$

$$\frac{r}{R} = \frac{2}{3}$$

$$\frac{r}{R-r} = \frac{2}{3-2} = \frac{2}{1}$$

$$\frac{R-r}{r} = \frac{1}{2}$$

Q2.

Volume of the hemisphere is $\frac{2}{3}\pi r^3$

Volume of biggest sphere r $= \frac{4}{3}\pi\left(\frac{r}{2}\right)^3 = \frac{1}{6}\pi r^3$

$\therefore$ Required ratio $= \frac{\frac{2}{3}\pi r^3}{\frac{1}{6}\pi r^3} = \frac{4}{1} = 4:1$

Q3.

The volume of a cone is given by:

$$V = \frac{1}{3}\pi r^2 h$$

- The larger cone has height $\frac{3h}{4}$.
- The smaller cone has height $\frac{h}{4}$.
- The bases of these cones are circular, and the radii of their bases must be considered.

Since both cones are similar (because they share the same shape and proportion), the ratio of their linear dimensions (radius and height) is the same.

Since the cones are similar, the ratio of their volumes is given by the cube of the ratio of their heights:

$$\text{Ratio of volumes} = \left(\frac{\text{height of larger cone}}{\text{height of smaller cone}}\right)^3$$

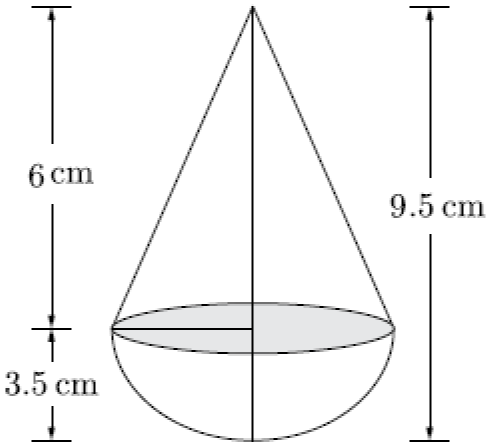
Substituting the given values:

$$\left(\frac{\frac{3h}{4}}{\frac{h}{4}}\right)^3 = \left(\frac{3}{1}\right)^3 = 3^3 = 27$$

Thus, the ratio of the volume of the larger cone to the smaller cone is **27:1**.

Q4.

As per question the figure is shown below. Here total volume of the toy is equal to the sum of volume of hemisphere and cone.



Volume of toy,

$$\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 (h + 2r)$$

$$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times (6 + 2 \times 3.5)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times (6 + 7)$$

$$= \frac{1}{3} \times \frac{22}{2} \times 3.5 \times 13$$

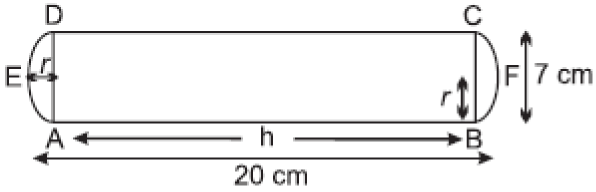
$$= \frac{1}{3} \times 11 \times 3.5 \times 13$$

$$= \frac{500.5}{3} = 166.83 \text{ cm}^3 \qquad \text{(Approx)}$$

Hence, the volume of the solid is 166.83 cm^3.

Q5.

ABCD is a cylinder and BFC and AED are two hemisphere which has radius $(r) = \frac{7}{2}$ cm



Hence, $AB = 20 - 2 \times \frac{7}{2}$

$\Rightarrow \qquad h = 13 \text{ cm}$

Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 13$$

$$= \frac{11 \times 13 \times 7}{2}$$

$$= \frac{1001}{2}$$

$$= 500.5 \text{ cm}^3$$

Volume of two hemisphere $= 2 \times \frac{2}{3} \pi r^3$

$$= 2 \times \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{49 \times 11}{3}$$

$$= \frac{539}{3}$$

$$= 179.67 \text{ cm}^3$$

Total volume of solid

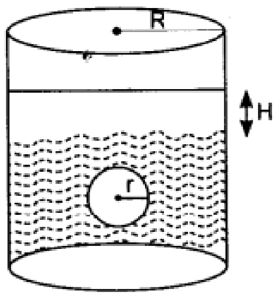
= Volume of two hemisphere

+ Volume of cylinder

$= 179.67 + 500.5$

$= 680.17 \text{ cm}^3$

Q6.



Diameter of sphere is 6 cm, so radius of sphere i.e. r =6/2 = 3 cm Diameter of the cylinder is 12 cm.

∴ Radius (R) =12/2 = 6 cm

Let the height of cylinder is H after increasing the water level.

Now, A.T.Q., Volume of sphere = Volume of cylinder

$4/3\pi r^3=\pi R^2H$

$4/3 \times (3)^3=(6)^2.H$

$H=4x3x3x3/3x6x6=1cm$

Water level will be raised by 1 cm.

Q7.

Solution:

Solid sphere: Radius, R = 10.5 cm

Cone: Radius, r = 3.5 cm and height, h = 3 cm.

Let solid sphere be melted and recast into 'n' number of smaller identical cones.

Then, Volume of 'n' cones = Volume of solid sphere

$$\Rightarrow \quad n \times \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi R^3$$

$$\Rightarrow \quad \frac{1}{3} \times n \times 3.5 \times 3.5 \times 3 = \frac{4}{3} \times 10.5 \times 10.5 \times 10.5$$

$$\Rightarrow \quad n = \frac{4 \times 10.5 \times 10.5 \times 10.5}{3 \times 3.5 \times 3.5} = 126$$

Hence, number of cones so formed = 126.

Q8.

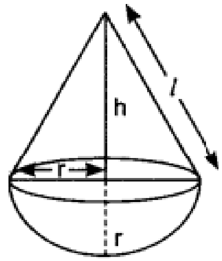
Solution:

Let radius of the base $= r$

Heigth of conical part $= h$

Slant height of conical part $= l$

$\therefore \quad l = \sqrt{h^2+r^2} \quad ...(i)$



Curved surface area of hemisphere = Curved surface area of cone

Here, $2\pi r^2 = \pi r l$

$\Rightarrow \quad l = 2r \quad ...(ii)$

From (*i*) & (*ii*), we have

$$2r = \sqrt{h^2+r^2}$$

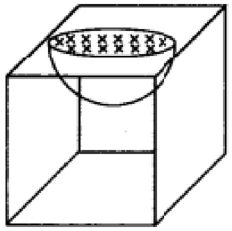
$$\Rightarrow \quad 4r^2 = h^2 + r^2 \Rightarrow h^2 = 3r^2$$

$$\Rightarrow \quad \frac{r^2}{h^2} = \frac{1}{3} \Rightarrow \frac{r}{h} = \frac{1}{\sqrt{3}}$$

$\therefore$ Required ratio of radius and height of conical part $= 1 : \sqrt{3}$

Q9.

Solution:



Edge of the cube=7 cm

Diameter of hemisphere =7 cm

Now,surface area of remaining solid=surface area of 6 faces of cube+surface area of hemisphere-surface area of circular top of diameter 7 cm

$6(\text{edge})^2+2\pi r^2-\pi r^2$

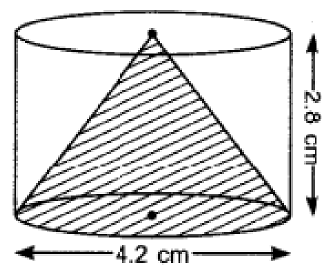
$=6(7)^2+2\pi r(7/2)^2-\pi(7/2)^2$

$=[6 \times 49+22/7 \times 49/4]\ \text{cm}^2 =[294 +38.5]\text{cm}^2 =332.5\ \text{cm}^2$

Q10.

Solution:

The shaded conical cavity is hollowed out.



For cylinder:

Radius of base, $r = \frac{4.2}{2} = 2.1$ cm

height, $h = 2.8$ cm

For cone:

Radius of base, $r = \frac{4.2}{2} = 2.1$ cm

height, $h = 2.8$ cm

slant height, $l = \sqrt{(2.1)^2 + (2.8)^2} = \sqrt{4.41 + 7.84}$ $[\because \; l^2 = r^2 + h^2]$

$= \sqrt{12.25} = 3.5$ cm.

Now, Total surface area of remaining solid

= Curved surface area of cylinder + Area of top circular base + Curved surface area of cone

$= 2\pi rh + \pi r^2 + \pi rl = \pi r[2h + r + l]$

$= \pi \times (2.1)[2 \times 2.8 + 2.1 + 3.5]$ cm^2

$= \frac{22}{7} \times 2.1 \times (5.6 + 5.6) = \frac{22}{7} \times 2.1 \times 11.2$ cm^2 $= 73.92$ cm^2