Chapter-Surface area and volume

Q1.
Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes?

Q2.
Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere?

Q3.
The volume of a right circular cylinder with its height equal to the radius is **25 1/7 cm³**. *Find the height of the cylinder. (Use π = 22/7)*

Q4.
A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in making of toy is 166 x 5/6 cm3 Find the height of the toy. Also, find the cost of painting the hemispherical part of the toy at the rate of 10 rupees per cm2

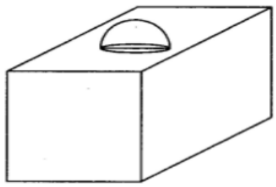
Q5.
From a solid right circular cylinder of height **14 cm** and base radius **6 cm**, a right circular Cone of same height and same base radius is removed. Find the volume of the remaining solid

Q6.
From a solid right circular cylinder of height 2.4 cm and radius 0.7 cm, a right circular cone of same height and same radius is cut out. Find the total surface area of the remaining solid

Q7.
A metallic solid sphere of radius 10.5 cm is melted and recasted into smaller solid cones, each of radius 3.5 cm and height 3 cm. How many cones will be made?

Q8.

In Figure, a decorative block, made up of two solids a cube and a hemisphere. The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has a diameter of 3.5 cm. Find the total surface area of the block.(use π=22/7)



Q9.

The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder, (use π=22/7)

Q10.

A toy is in the form of a cone of base radius 3.5 cm mounted on a hemisphere of base diameter 7 cm. If the total height of the toy is 15.5 cm, find the total surface area of the toy(use π =22/7)

Solution 1.

Solution : We have, $\frac{h_1}{h_2} = \frac{1}{3}$ and $\frac{r_1}{r_2} = \frac{3}{1}$

Now, Ratio of volumes $= \frac{V_1}{V_2}$

$$= \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

$$= \left(\frac{r_1}{r_2}\right)^2 \left(\frac{h_1}{h_2}\right)$$

$$= \left(\frac{3}{1}\right)^2 \times \left(\frac{1}{3}\right)$$

$$= 9 \times \frac{1}{3} = \frac{3}{1}$$

$\therefore$ The ratio of their volumes is 3 : 1. **Ans.**

Solution 2.

Let radius of hemisphere be r units

Volume of hemisphere = S.A. of hemisphere

$$\frac{2}{3}\pi r^3 = 3\pi r^2$$

$$\Rightarrow \quad r = \frac{9}{2} \text{ or diameter} = 9 \text{ units}$$

Solution 3.

Given :

Height of cylinder = Radius of cylinder

$\Rightarrow \quad h = r$

And, Volume of cylinder $= 25\frac{1}{7}\ \text{cm}^3 = \frac{176}{7}\ \text{cm}^3$

We know, Volume of cylinder $= \pi r^2 h$

$= \pi h^3 \qquad [\because h = r]$

$\Rightarrow \quad \frac{176}{7} = \frac{22}{7} h^3$

$\Rightarrow \quad h^3 = 8$

$\Rightarrow \quad h = 2.$

Hence, the height of the cylinder is 2 cm. **Ans.**

Solution 4.

Hemisphere: Radius of hemisphere, $r = 3.5$ cm

Cone: Radius of cone, $r = 3.5$ cm and let height of cone be 'h'. Then,

Volume of toy = Volume of cone + Volume of hemisphere

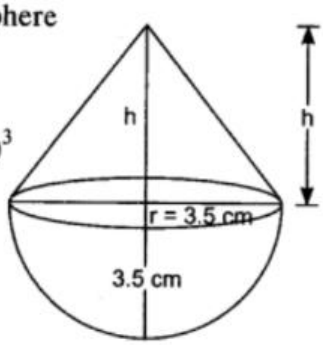
$$166\frac{5}{6} = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$\Rightarrow \quad 166\frac{5}{6} = \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 h + \frac{2}{3} \times \frac{22}{7} \times (3.5)^3$$

$$\Rightarrow \quad \frac{1001}{6} = \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 (h + 2 \times 3.5)$$

$$\Rightarrow \quad \frac{1001}{6} = \frac{22 \times 3.5 \times 3.5}{3 \times 7} (h + 7)$$

$$\Rightarrow \quad (h + 7) = \frac{1001 \times 3 \times 7}{22 \times 3.5 \times 3.5 \times 6} = 13 \quad \Rightarrow \quad h = 13 - 7 = 6 \text{ cm}$$



$\therefore$ Height of cone, $h = 6$ cm

$\therefore$ Height of toy $= h + r = 6 + 3.5 = 9.5$ cm

Curved surface area of hemisphere $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times (3.5)^2 = 77\ \text{cm}^2$$

Cost of painting hemispherical part of toy = ₹ 10 per cm^2

$\therefore$ Total cost of painting hemispherical portion of toy = ₹ (10×77) = ₹ 770.

Solution 5.

Given : Height of cylinder (h) = 14 cm

And, Base radius of cylinder (r) = 6 cm

Radius and height of right circular cone is same as that of the cylinder.

$\therefore$ Height of cone = 14 cm

and radius of cone = 6 cm

Now, Volume of remaining solid

= Volume of cylinder – Volume of cone

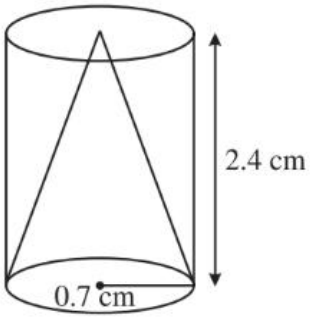
$= \pi r^2 h - \frac{1}{3}\pi r^2 h$

$= \frac{2}{3}\pi r^2 h$

$= \frac{2}{3} \times \frac{22}{7} \times 6^2 \times 14$

$= 1056 \text{ cm}^3$ **Ans.**

$= 2.5$ cm



Total surface area of the remaining solid

= CSA of cylinder + CSA of cone + Area of top

$= 2\pi rh + \pi rl + \pi r^2$

$= \pi r\,[2h + l + r]$

$= \frac{22}{7} \times 0.7\,[2 \times 2.4 + 2.5 + 0.7]$

$= 2.2\,[4.8 + 2.5 + 0.7]$

$= 2.2 \times 8 = 17.6 \text{ cm}^2$

Solution 7.

Volume of metal in cones = Volume of solid sphere

Let n = number of cones

$n \times$ volume of each cone = volume of solid sphere

$$n = \frac{\text{Volume of sphere}}{\text{Volume of cone}}$$

$$= \frac{\frac{4}{3}\pi r_{sp}^3}{\frac{1}{3}\pi r_{\text{cone}}^2 h}$$

$$= \frac{4\times 10.5\times 10.5\times 10.5}{3.5\times 3.5\times 3}$$

$$= \frac{4\times 105\times 105\times 105\times 10\times 10}{35\times 35\times 3\times 10\times 10\times 10}$$

$$= \frac{4\times 3\times 105}{10} = 126$$

So, 126 cones will be made.

Solution 8.

Side of the cube = 6 cm

Total surface area of cube = $6 \times (\text{side})^2 = 6 \times (6)^2 = 216\ \text{cm}^2$

Area covered on the face of cube by circular part of hemisphere = $\pi r^2 = \frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2}\ \text{cm}^2$

Curved surface area of hemisphere = $2 \times \pi \times r^2 = 2\times\frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2}\ \text{cm}^2$

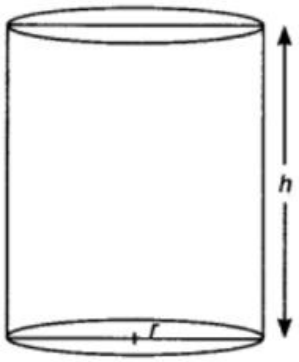
So, Total surface area of the block = Surface area of cube – Area of circular face of hemisphere + Curved surface area of hemisphere

$$= 216 - \frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2} + 2\times\frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2}$$

$$= 216 + \frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2} = 216 + 9.625 = 225.625\ \text{cm}^2$$

Solution 9.

Here $r + h = 37$ [Given, where $r \rightarrow$ radius, $h \rightarrow$ height]



$$\text{Total surface area of cylinder} = 2\pi r(h + r) = 2\pi rh + 2\pi r^2$$

$$\Rightarrow \quad 2\pi r(h + r) = 1628$$

$$\Rightarrow \quad 2\pi r \times 37 = 1628$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times 37 = 1628$$

$$\Rightarrow \quad r = \frac{1628 \times 7}{2 \times 22 \times 37} = 7 \text{ cm}$$

Given, $r + h = 37$

$$7 + h = 37$$

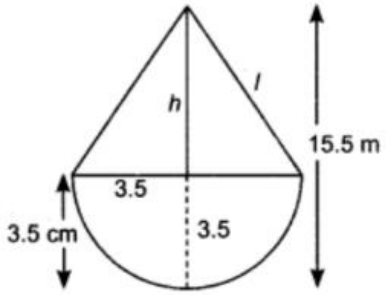
$$\Rightarrow \quad h = 37 - 7 = 30 \text{ cm}$$

Hence, volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 30 = 4620 \text{ cm}^3$$

Solution 10.

Here, given that $h = 15.5 - 3.5 = 12$ cm



Also, slant height of cone, $l = \sqrt{h^2 + r^2}$

$$= \sqrt{(12)^2 + (3.5)^2} \text{ cm}$$

$$= \sqrt{144 + 12.25} \text{ cm}$$

$$= \sqrt{156.25} \text{ cm} = 12.5 \text{ cm}$$

$\therefore$ Curved Surface Area of cone $= \pi rl = \frac{22}{7} \times 3.5 \times 12.5$

$$= 137.5 \text{ cm}^2$$

$\therefore$ Surface area of hemisphere $= 2\pi r^2 = 2 \times \frac{22}{7} \times (3.5)^2 = 77 \text{ cm}^2$

Hence, Total Surface Area of toy = Surface area of hemisphere + Curved Surface Area of cone

$$= 77 + 137.5 = 241.5 \text{ cm}^2$$