Chapter-Some Applications of Trigonometry

Q1.

A round balloon of radius r subtends an angle α at the eye of the observer, while the angle of elevation of its centre is β. Prove that the height of the centre of the balloon is $r \sin\beta \csc\frac{\alpha}{2}$.

Q2.

If the angle of elevation of a cloud from a point h meter above a lake has measure α and the angle depression of its reflection of in the lake has measureβ . Prove that the height of the cloud is $\dfrac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha}$

Q3.

From an aeroplane vertically above a straight horizontal road, the angles of depression of two consecutive mile stones on opposite sides of the aeroplane are observed to be α and β. Show that the height in miles of aeroplane above the road is given by

$$\frac{\tan\alpha \tan\beta}{\tan\alpha + \tan\beta}$$

Q4

Two stations due South of a leaning tower which leans towards the North, are at distances a and b from its foot. If α and β are the elevations of the top of the tower from these stations, then prove that its inclination θ to the horizontal is given by

$$\cot\theta = \frac{b\cot\alpha - a\cot\beta}{b - a}$$

Q5

From a point on the ground the angle of elevation of top of a tower is α . On moving 'a' meter towards the tower, the elevation changes to β. Prove that he height of the tower is $\dfrac{a\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}$

Q6.

If the angles of elevation of the top of a tower from two points distant a and $b(a > b)$ from its foot and in the same straight line from it are respectively 30° and 60°, then find the height of the tower.

Q7.

A boy standing on a horizontal plane finds a bird, flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of a 20 m high building finds the angle of elevation of the same bird to be 45°. Both the boy and the girl are on the opposite side of the bird. Find the distance of the bird from the girl.

Q8.

A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. 10 seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Q9.

A ladder rests against a wall at an angle a to the horizontal. Its foot is pulled away from the B wall through a distance p, so that it slides a distance q down the wall making an angle β with the horizontal. Prove that :

$$\frac{p}{q} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$$

Q10.

From a window (9 m above the ground) of a house in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are 30° and 60° respectively. Find the height of the opposite house and the width of the street

Solutions

Q1.

Let O be the centre of the balloon and P be the eye of the observer

And $\angle APB$ be the angle subtended by the balloon

$\therefore \angle APB = \alpha$

$\therefore \angle APO = \angle BPO = \frac{\alpha}{2}$

In $\triangle OAP$, $\sin\frac{\alpha}{2} = \frac{OA}{OP}$

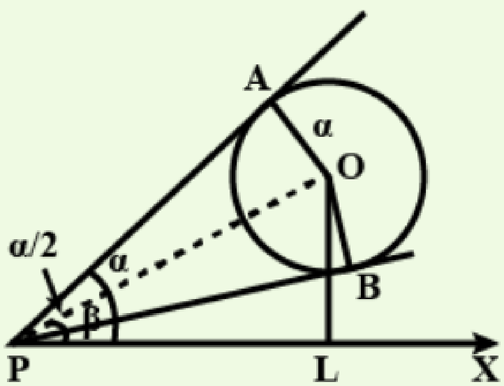
$\Rightarrow \sin\frac{\alpha}{2} = \frac{r}{OP}$

$\Rightarrow OP = r \operatorname{cosec}\frac{\alpha}{2} - (1)$

In $\triangle OPC$

$\sin\beta = \frac{OL}{OP}$ or $OL = OP \sin\beta$

$\therefore OL = r \operatorname{cosec}\frac{\alpha}{2}\sin\beta$



Q2.

Let AB be the surface of the lake and let P be a point of observation such that AP = h metres.

Let C be the position of the cloud and C' be its reflection in the lake.

Then, CB=C'B. Let PM be perpendicular from P on CB. Then, ∠CPM= α and ∠ MPC' = β. Let CM = x.

Then, CB=CM+MB=CM+PA = x+h

In △ CPM, we have,

$$\tan\alpha = \frac{CM}{PM}$$

$$\tan\alpha = \frac{x}{AB}$$

$$AB = x\cot\alpha \quad .....(1)$$

In △PMC', we have

$$\tan\beta = \frac{C'M}{PM}$$

$$\tan\beta = \frac{x+2h}{AB}$$

$$AB = (x+2h)\cot\beta \quad .....(2)$$

From (1) & (2), we have,

$$x\cot\alpha = (x+2h)\cot\beta$$

$$x(\cot\alpha - \cot\beta) = 2h\cot\beta$$

$$x\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right) = \frac{2h}{\tan\beta}$$

$$x\left(\frac{\tan\beta - \tan\alpha}{\tan\alpha\tan\beta}\right) = \frac{2h}{\tan\beta}$$

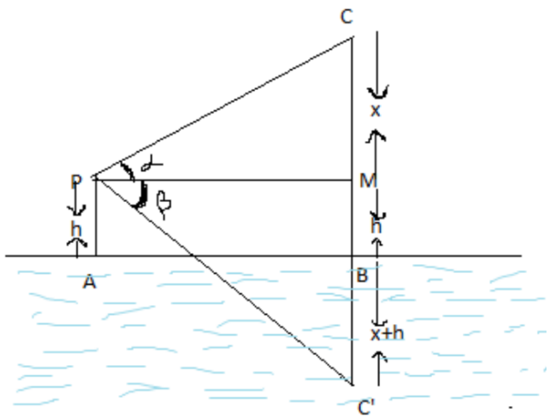
$$x = \frac{2h\tan\alpha}{\tan\beta - \tan\alpha}$$

Hence, the height CB of the cloud is given by

$$CB = x + h$$

$$CB = \frac{2h\tan\alpha}{\tan\beta - \tan\alpha} + h$$

$$CB = \frac{2h\tan\alpha + h\tan\beta - h\tan\alpha}{\tan\beta - \tan\alpha} = \frac{h(\tan\alpha + \tan\beta)}{\tan\beta - \tan\alpha}$$



Q3.

Let B and C be the two consecutive mile stones.

$\therefore$ BC = BD + CD = 1 miles

Let the height of the aeroplane AD = H miles

$\Rightarrow$ In $\triangle ABD$, $\tan\alpha = \frac{AD}{BD}$........ $\tan(\Theta) = \frac{\text{Opposite}}{\text{Adjacent}}$

$\Rightarrow \tan\alpha = \frac{h}{BD}$

$\Rightarrow BD = \frac{h}{\tan\alpha}$--------(1)

$\Rightarrow$ In $\triangle ACD$, $\tan\beta = \frac{AD}{CD}$

$\Rightarrow \tan\beta = \frac{h}{CD}$.......... $\tan(\Theta) = \frac{\text{Opposite}}{\text{Adjacent}}$

$\Rightarrow CD = \frac{h}{\tan\beta}$--------(2)

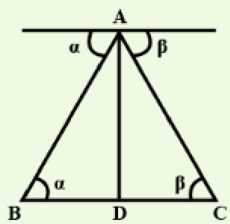
$\Rightarrow BC = BD + CD = \frac{h}{\tan\alpha} + \frac{h}{\tan\beta}$

$\Rightarrow BC = h\left(\frac{1}{\tan\alpha} + \frac{1}{\tan\beta}\right)$

$\Rightarrow 1 = h\left(\frac{\tan\alpha + \tan\beta}{\tan\alpha\tan\beta}\right)$

$\therefore h = \frac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$

$\Rightarrow$ Hence, height of the aeroplane is $\frac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$



Q4

height of the tower DE = h

Distance between first station to foot of tower AD = a + x

Distance between second station to foot of tower BD = b + x

Distance between C and D = x

Give α, β are the angle of elevation two stations to top of the tower

thatis∠DAE = α, ∠DBE = β, ∠DCE = θ

InΔADE

$\text{Cot}\theta = \frac{x}{h} ------------------\rightarrow (1)$

InΔBDE

$\text{Cot}\beta = \frac{b+x}{h}$

$(b + x) = h\text{Cot}\beta$ (multiply a on both sides)

$(ab + ax) = ha\text{Cot}\beta ------------------\rightarrow (2)$

InΔCDE

$\text{Cot}\alpha = \frac{a+x}{h}$

$(a + x) = h\text{Cot}\alpha$ (multiply b on both sides)

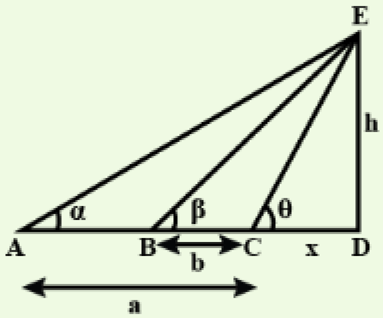
$(ab + bx) = hb\text{Cot}\alpha ------------------\rightarrow (3)$

substract(3) – (2)

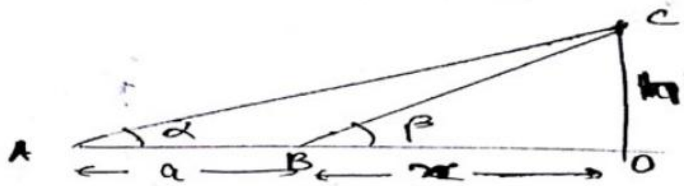
$(b - a)x = h(b\text{Cot}\alpha - a\text{Cot}\beta)$

$\frac{x}{h} = \frac{b\text{Cot}\alpha - a\text{Cot}\beta}{b-a}$

$\text{Cot}\theta = \frac{b\text{Cot}\alpha - a\text{Cot}\beta}{b-a}.$



Q5



In Δ AOC,

$\tan\alpha = \frac{h}{a+x}$ $\Rightarrow$ $(a+x)\tan\alpha = h$.

$\Rightarrow x = \frac{h - a\tan\alpha}{\tan\alpha}$. —— (1)

In Δ BOC,

$\tan\beta = \frac{h}{x}$ $\Rightarrow$ $x = \frac{h}{\tan\beta}$. —— (2)

from (1) & (2).

$\frac{h}{\tan\beta} = \frac{h}{\tan\alpha} - a$

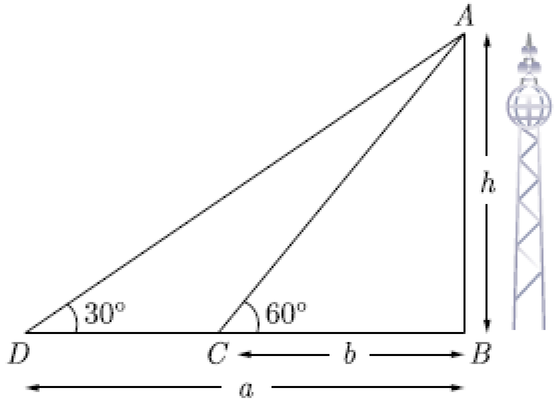
$h\left[\frac{1}{\tan\beta} - \frac{1}{\tan\alpha}\right] = -a$

$h = \frac{a}{\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right)}$

$$\boxed{h = \frac{a\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}}$$

Q6.

Let the height of tower be h. As per given in question we have drawn figure below.



From ΔABD, $\frac{h}{a} = \tan 30^\circ$

$$h = a \times \frac{1}{\sqrt{3}} = \frac{a}{\sqrt{3}} \qquad ...(1)$$

From ΔABC, $\frac{h}{b} = \tan 60^\circ$

$$h = b \times \sqrt{3} = b\sqrt{3} \qquad ...(2)$$

From (1) we get $a = \sqrt{3}\,h$

From (2) get $b = \frac{h}{\sqrt{3}}$

Thus $a \times b = \sqrt{3}\,h \times \frac{h}{\sqrt{3}}$

$$ab = h^2$$

$$h = \sqrt{ab}$$

Hence, the height of the tower is $\sqrt{ab}$.

Q7.

Let A be the position of the bird and E and C be the positions of the girl and the boy, respectively.

Then,

$\angle ACB = 30^\circ, \angle AED = 45^\circ, AC = 100\text{ m}, EF = 20\text{ m}$

In right $\triangle ACB$,
we have,
$\sin 30^\circ = \frac{AB}{AC}$

$\frac{1}{2} = \frac{AB}{100}$

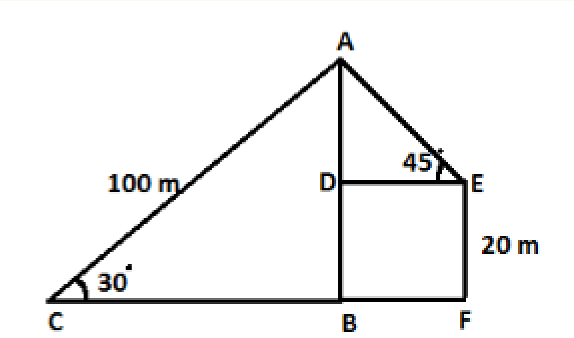
$AB = 50\text{ m}$
$AD = AB - BD$
$AD = 50 - EF$
$AD = 50 - 20 = 30\text{ m}$

In right $\triangle ADE$, we have,
$\sin 45^\circ = \frac{AD}{AE}$

$\frac{1}{\sqrt{2}} = \frac{30}{AE}$

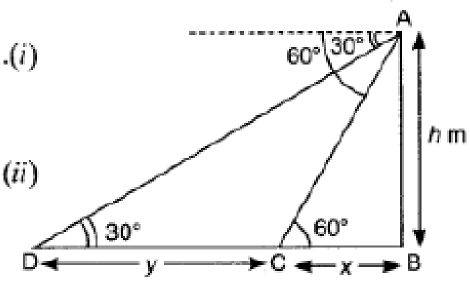
$AE = 30\sqrt{2} = 42.3\text{ m}$

Therefore, the distance of bird from the girl is 42.3 m.



Q8.

Solution:



In ΔABC, $\frac{AB}{BC} = \tan 60° \Rightarrow \frac{h}{x} = \sqrt{3}$

$\Rightarrow h = \sqrt{3}x$ ...(*i*)

In ΔABD, $\frac{AB}{BD} = \tan 30°$

$\Rightarrow \frac{h}{x+y} = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{x+y}{\sqrt{3}}$ ...(*ii*)

From (*i*) and (*ii*), we get

$$\sqrt{3}x = \frac{x+y}{\sqrt{3}}$$

$\Rightarrow 3x = x + y \Rightarrow 2x = y \Rightarrow x = \frac{y}{2}$

It is given that car covers a distance of y in 10 seconds. So, in order to cover the distance $x = \frac{y}{2}$, car will take 5 seconds. So, total time taken by the car to reach the foot of the tower is 15 seconds.

Q9.

Let AB is a wall and AC is a ladder with length x. The foot of ladder is pulled through distance p so that it slides a distance q down the wall.

$\angle ACB = \alpha$ and $\alpha EDB = \beta$

From right angled ΔABC,

$$\sin\alpha = \frac{AB}{AC}$$

$$\sin\alpha = \frac{AB}{x}$$

$AB = x\sin\alpha$ .....(i)

and $\cos\alpha = \dfrac{BC}{AC}$

$$\cos\alpha = \frac{BC}{x}$$

$BC = x\cos\alpha$ ....(ii)

$BD = BC + CD$

$= x\cos\alpha + p$

$BE = AB – AE$

$= x\sin\alpha – q$ ....(iii)

From right angled ΔEBD,

$$\sin\beta = \frac{BE}{DE}$$

$$\sin\beta = \frac{x\sin\alpha - q}{x}$$

[$\because$ From equation (iii), $BE = \sin x$ (length of ladder)]

$$\sin\beta = \sin\alpha - \frac{q}{x}$$

$$\frac{q}{x} = \sin\alpha - \sin\beta$$

$q = x(\sin\alpha - \sin\beta)$.............(iv)

$$\cos\beta = \frac{BD}{DE}$$

$$\frac{x\cos\alpha - P}{x}$$

[$\because BD = x\cos\alpha + p$]

$$\cos\beta = \cos\alpha + \frac{P}{x}$$

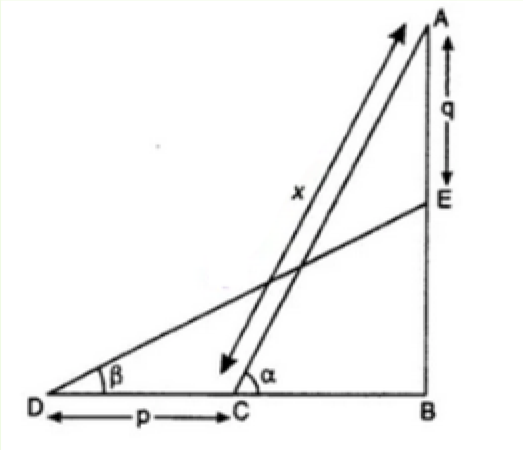
$$\frac{P}{x} = \cos\beta - \cos\alpha$$

$P = x(\cos\beta - \cos\alpha)$.............(v)

Divide equation (v) by equation (iv)

$$\frac{p}{q} = \frac{x(\cos\beta - \cos\alpha)}{x(\sin\alpha - \sin\beta)}$$

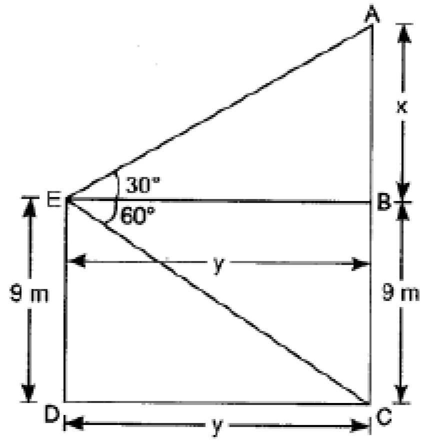
$$\frac{p}{q} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$$



Q10.

Let ED be the window of height 9 m and AC be house of height $x + 9$. DC is the street of width y.

Here, $\angle AEB = 30^\circ$ and $\angle CEB = 60^\circ$



In ΔCBE, $\frac{CB}{BE} = \tan 60^\circ$

$\Rightarrow \quad \frac{9}{y} = \sqrt{3}$

$\Rightarrow \quad y = \frac{9}{\sqrt{3}} = \frac{9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{9\sqrt{3}}{3} = 3\sqrt{3}\,\text{m} \quad \ldots(i)$

In ΔABE, $\frac{AB}{BE} = \tan 30^\circ \Rightarrow \frac{x}{y} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \frac{x}{3\sqrt{3}} = \frac{1}{\sqrt{3}} \quad$ [From (i)]

$x = 3\,\text{m}$

Height of the house $= x + 9 = 3 + 9 = 12$ m

Width of the street $= 3\sqrt{3} = 3 \times 1.732 = 5.196$ m