Chapter-Real numbers
Question bank

Q1. Find the HCF of 54 and 90 using prime factorization.

Q2. Find the LCM of 36 and 48 using prime factorization.

Q3. Prove that $\sqrt{5}$ is irrational.

Q4. If HCF(180, 168) = 12, find LCM(180, 168).

Q5. Find the HCF of 65 and 117 using the prime factorization method.

Q6. Prove that $7 + \sqrt{3}$ is irrational.

Q7. Find the HCF and LCM of 24, 36, and 40.

Q8. Is the product of a non-zero rational and an irrational number always irrational? Justify.

Q9. Prove that $7\sqrt{3}$ is irrational.

Q10. The LCM of two numbers is 120 and their HCF is 10. If one of the numbers is 60, find the other number.

Solutions

Solution 1:

$54 = 2 \times 3^3$

$90 = 2 \times 3^2 \times 5$

HCF $= 2 \times 3^2 = 18$

Solution 2:

$36 = 2^2 \times 3^2$

$48 = 2^4 \times 3$

LCM $= 2^4 \times 3^2 = 144$

Solution 3:

Assume $\sqrt{5}$ is rational.

Then $\sqrt{5} = a/b$, where a and b are coprime positive integers.

Squaring both sides: $5 = a^2/b^2$

Cross multiplying: $5b^2 = a^2$

This means a^2 is divisible by 5, so a must be divisible by 5.

Let $a = 5k$, where k is an integer.

Substituting: $5b^2 = (5k)^2 = 25k^2$

Simplifying: $b^2 = 5k^2$

This means b^2 is divisible by 5, so b must be divisible by 5.

But if both a and b are divisible by 5, they are not coprime.

This contradicts our initial assumption.

Solution 4:

HCF × LCM = Product of numbers

$12 \times \text{LCM} = 180 \times 168$

LCM $= (180 \times 168) / 12 = 2520$

Solution 5:
$65 = 5 \times 13$
$117 = 3^2 \times 13$
HCF = 13

Solution 6: Assume that $7 + \sqrt{3}$ is rational.

If it's rational, it can be expressed as a fraction p/q, where p and q are integers and $q \neq 0$. So, we can write: $7 + \sqrt{3} = p/q$

Subtracting 7 from both sides: $\sqrt{3} = p/q - 7$

$\sqrt{3} = (p - 7q)/q$

This implies that $\sqrt{3}$ is rational, as it is expressed as a fraction of integers.

However, we know that $\sqrt{3}$ is irrational (this can be proven separately, similar to the proof for $\sqrt{2}$).

This is a contradiction. Our assumption in step 1 must be false.

Therefore, $7 + \sqrt{3}$ must be irrational.

Solution 7:
$24 = 2^3 \times 3$
$36 = 2^2 \times 3^2$
$40 = 2^3 \times 5$
HCF = $2^2 = 4$
LCM = $2^3 \times 3^2 \times 5 = 360$

Solution 8: Yes. If r ($\neq 0$) is rational and i is irrational, ri must be irrational.

Solution 9: Assume that $7\sqrt{3}$ is rational.

If it's rational, it can be expressed as a fraction p/q, where p and q are integers and $q \neq 0$. So, we can write: $7\sqrt{3} = p/q$

Dividing both sides by 7: $\sqrt{3} = p/(7q)$

Let's call $7q = r$, so we have: $\sqrt{3} = p/r$, where p and r are integers.

Squaring both sides: $3 = p^2/r^2$

Cross-multiplying: $3r^2 = p^2$

This means p^2 is divisible by 3, so p must be divisible by 3.

Let $p = 3k$, where k is an integer.

Substituting back: $3r^2 = (3k)^2 = 9k^2$

Simplifying: $r^2 = 3k^2$

This means r^2 is divisible by 3, so r must be divisible by 3.

But if both p and r are divisible by 3, they are not coprime, contradicting the assumption that p/r is in its lowest terms.

This contradiction means our initial assumption must be false.

Therefore, $7\sqrt{3}$ must be irrational.

Solution 10: LCM × HCF = Product of the two numbers

$120 \times 10 = 60 \times b$, where b is the unknown number

$1200 = 60b$

$b = 1200 \div 60$

$b = 20$

The other number is 20.