Chapter- Real Numbers

Rational Numbers: A number in the form p/q , where p and q are co-prime numbers and $q \neq 0$, is known as a rational number.

Irrational Numbers: A number is called irrational if it cannot be written in the form p/q , where p and q are integers and $q \neq 0$.

Fundamental Theorem of Arithmetic

The Fundamental Theorem of Arithmetic states that every positive integer greater than 1 can be expressed uniquely as a product of prime factors, up to the order of the factors.

Key Points:

1. Every number greater than 1 is either a prime number or can be factored into prime numbers.
2. The factorization is unique, meaning there's only one way to express the number as a product of primes.
3. The order of the factors doesn't matter (e.g., $2 \times 3 = 3 \times 2$).

Example 1:

Express 84 as a product of primes.

Solution:

$84 \div 2 = 42$

$42 \div 2 = 21$

$21 \div 3 = 7$

Therefore, $84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$

Example 2:

Find the prime factorization of 360.

Solution:

$360 \div 2 = 180$

$180 \div 2 = 90$

$90 \div 2 = 45$

$45 \div 3 = 15$

$15 \div 3 = 5$

Therefore, $360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 2^3 \times 3^2 \times 5$

Applications:

1. Finding HCF (Highest Common Factor): Take common prime factors with the lowest power.
2. Finding LCM (Least Common Multiple): Take all prime factors with the highest power.

   Example:

   Find the HCF and LCM of 48 and 180.

   Solution:

   $48 = 2^4 \times 3$

   $180 = 2^2 \times 3^2 \times 5$

   $HCF = 2^2 \times 3 = 12$

   $LCM = 2^4 \times 3^2 \times 5 = 720$

Proof of Irrationality

These proofs use the method of contradiction, which assumes the opposite of what we want to prove and then shows that this assumption leads to a contradiction.

General Proof Structure:

1. Assume the number is rational.
2. Express it as a fraction a/b where a and b are coprime integers.
3. Square both sides of the equation.
4. Show that this leads to a contradiction (usually that both a and b must be divisible by the number under the square root).
5. Conclude that the original assumption must be false, so the number is irrational.

   Example:

   We will prove that $\sqrt{2}$ is irrational using the method of contradiction.

Proof:

1. Assume √2 is rational.
2. Then √2 can be expressed as a/b, where a and b are coprime positive integers.
3. Squaring both sides: $2 = a^2/b^2$
4. Cross multiplying: $2b^2 = a^2$
5. This means a^2 is even, so a must be even.
6. Let a = 2k, where k is an integer.
7. Substituting: $2b^2 = (2k)^2 = 4k^2$
8. Simplifying: $b^2 = 2k^2$
9. This means b^2 is even, so b must be even.
10. But if both a and b are even, they are not coprime.
11. This contradicts our initial assumption. Therefore, √2 must be irrational.

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