Chapter- Quadratic Equations

1. Standard Form of a Quadratic Equation

A quadratic equation is an equation of the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$.

Example 1:

Identify if $2x^2 - 5x + 3 = 0$ is a quadratic equation.

Solution: Yes, it is a quadratic equation with a = 2, b = -5, and c = 3.

Example 2:

Express $3(x - 1)^2 - 4 = 0$ in standard form.

Solution:

$3(x^2 - 2x + 1) - 4 = 0$

$3x^2 - 6x + 3 - 4 = 0$

$3x^2 - 6x - 1 = 0$

2. Solutions of Quadratic Equations

a. Factorization Method

Example 1:

Solve $x^2 - 7x + 12 = 0$ by factorization.

Solution:

$x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

$x = 3$ or $x = 4$

Example 2:

Solve $2x^2 + 7x + 3 = 0$ by factorization.

Solution:

$2x^2 + 7x + 3 = 0$

$(2x + 1)(x + 3) = 0$

$x = -1/2$ or $x = -3$

b. Quadratic Formula

The quadratic formula for $ax^2 + bx + c = 0$ is:

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$

Example 1:

Solve $x^2 + 5x + 6 = 0$ using the quadratic formula.

Solution:

a = 1, b = 5, c = 6

$x = [-5 \pm \sqrt{(5^2 - 4(1)(6))}] / (2(1))$

$x = [-5 \pm \sqrt{(25 - 24)}] / 2$

x = [-5 ± 1] / 2

x = -3 or x = -2

Example 2:

Solve $2x^2 - 7x + 3 = 0$ using the quadratic formula.

Solution:

a = 2, b = -7, c = 3

$x = [7 \pm \sqrt{((-7)^2 - 4(2)(3))}] / (2(2))$

$x = [7 \pm \sqrt{(49 - 24)}] / 4$

x = (7 ± 5) / 4

x = 3 or x = 1/2

3. Nature of Roots

The nature of roots depends on the discriminant ($b^2 - 4ac$):

- If $b^2 - 4ac > 0$, there are two distinct real roots.
- If $b^2 - 4ac = 0$, there are two equal real roots.
- If $b^2 - 4ac < 0$, there are no real roots.

Example 1:

Determine the nature of roots for $x^2 - 4x + 4 = 0$.

Solution:

a = 1, b = -4, c = 4

$b^2 - 4ac = (-4)^2 - 4(1)(4) = 16 - 16 = 0$

The equation has two equal real roots.

Example 2:

Determine the nature of roots for $2x^2 + x + 1 = 0$.

Solution:

a = 2, b = 1, c = 1

$b^2 - 4ac = 1^2 - 4(2)(1) = 1 - 8 = -7$

The equation has no real roots.