Chapter-Quadratic Equations
Question bank

Q1.

.Solve for x : $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

Q2.

Solve for x : $4x^2 + 4bx - (a^2 - b^2) = 0$

Q3.

Solve, for x : $\sqrt{3}\,x^2 + 10x + 7\sqrt{3} = 0$

Q4.

).If the roots of the quadratic equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal, prove that $2a = b + c$.

Q5.

.If -3 is a root of quadratic equation $2x^2 + px - 15 = 0$, while the quadratic equation $x^2 - 4px + k = 0$ has equal roots. Find the value of k.

Q6.

).Solve the following quadratic equation for x :

$$9x^2 - 9(a+b)x + 2a^2 + 5ab + 2b^2 = 0$$

Q7.

The difference of squares of two numbers is 88. If the larger number is 5 less than twice the smaller number, then find the two numbers.

Q8.

A motor boat whose speed is 20 km/h in still water, takes 1 hour more to go 48 km upstream than to return downstream to the same spot. Find the speed of the stream.

Q9.

Find the value of m so that the quadratic equation mx(x – 7) + 49 = 0 has two equal roots.

Q10.

Some students planned a picnic. The total budget for food was rs 2,000. But 5 students failed to attend the picnic and thus the cost of food for each member increased by rs 20. How many students attended the picnic and how much did each student pay for the food.

Solution :

Solutions

Solution 1.

Ans :

We have $\frac{1}{a+b+x}-\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$

$$\frac{-(a+b)}{x^2+(a+b)x}=\frac{b+a}{ab}$$

$$x^2+(a+b)x+ab=0$$

$$(x+a)(x+b)=0$$

$$x=-a,x=-b$$

Hence $x=-a,-b$

Q2.

We have $4x^2+4bx-(a^2-b^2)=0$

Comparing with $Ax^2+Bx+C=0$ we get

$$A=4,B=4b \text{ and } C=b^2-a^2$$

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$=\frac{-4b\pm\sqrt{(4b)^2-4.4(b^2-a)}}{2.4}$$

$$=\frac{-4b\pm\sqrt{16b^2-16b^2+16a^2}}{8}$$

$$=\frac{-4b\pm 4a}{8}$$

$$=-\frac{(a+b)}{2},\frac{(a-b)}{2}$$

Hence the roots are $-\frac{(a+b)}{2}$ and $\frac{(a-b)}{2}$

Q3.

We have $\sqrt{3}\,x^2+10x+7\sqrt{3} = 0$

$$\sqrt{3}\,x^2+3x+7x+7\sqrt{3} = 0$$

$$(x+\sqrt{3})(\sqrt{3}\,x \quad 7 = 0$$

$$(x+\sqrt{3})(\sqrt{3}\,x \quad 7 = 0$$

$$x = -\sqrt{3} \text{ and } x = \frac{-7}{\sqrt{3}}$$

Hence roots $x = -\sqrt{3}$ and $x = \frac{-7}{\sqrt{3}}$

Q4.

We have $(a-b)x^2+(b-c)x+(c-a) = 0$

Comparing with $ax^2+bx+c = 0$ we get

$a = (a-b,),\ b = (b-c),\ c = c-a$

For real and equal roots, $b^2-4ac = 0$

$$(b-c)^2-4(a-b)(c-a) = 0$$

$$b^2+c^2-2bc-4(ac-a^2-bc+ab) = 0$$

$$b^2+c^2-2bc-4ac+4a^2+4bc-4ab = 0$$

$$4a^2+b^2+c^2+2bc-4ab-4ac = 0$$

Using $a^2+b^2+c^2+2ab+2bc+2ca = (a+b+c)^2$,

$$(-2a+b+c)^2 = 0$$

$$-2a+b+c = 0$$

Hence, $b+c = 2a$

Q5.

Given -3 is a root of quadratic equation.

We have $2x^2 + px - 15 = 0$

Since 3 is a root of above equation, it must satisfy it.

Substituting $x = 3$ in above equation we have

$$2(-3)^2 + p(-3) - 15 = 0$$

$$2 \times 9 - 3p - 15 = 0 \Rightarrow p = 1$$

Since $x^2 - 4px + k = 0$ has equal roots,

or $x^2 - 4x + k = 0$ has equal roots,

$$b^2 - 4ac = 0$$

$$(-4)^2 - 4k = 0$$

$$16 - 4k = 0$$

$$4k = 16 \Rightarrow k = 4$$

Q6.

We have $9x^2-9(a+b)x+2a^2+5ab+2b^2=0$

Now $2a^2+5ab+2b^2=2a^2+4ab+ab+2b^2$

$=2a[a+2b]+b[a+2b]$

$=(a+2b)(2a+b)$

Hence the equation becomes

$$9x^2-9(a+b)x+(a+2b)(2a+b)=0$$

$$9x^2-3[3a+3b]x+(a+2b)(2a+b)=0$$

$$9x^2-3[(a+2b)+(2a+b)]x+(a+2b)(2a+b)=0$$

$$9x^2-3(a+2b)x-3(2a+b)x+(a+2b)(2a+b)=0$$

$$3x[3x-(a+2b)]-(2a+b)[3x-(a+2b)]=0$$

$$[3x-(a+2b)][3x-(2a+b)]=0$$

$$3x-(2a+b)=0$$

$$x=\frac{a+2b}{3}$$

$$3x-(2a+b)=0$$

$$x=\frac{2a+b}{3}$$

Hence, roots are $\frac{a+2b}{3}$ and $\frac{2a+b}{3}$.

Q7.

Let smaller number be x and other larger number be $2x - 5$.

A.T.Q. $(2x-5)^2 - x^2 = 88 \Rightarrow 4x^2 - 20x + 25 - x^2 = 88$

$\Rightarrow \quad 3x^2 - 20x - 63 = 0$

$\therefore \quad$ Discriminant, $D = (-20)^2 - 4 \times 3 \times (-63) = 400 + 756 = 1156 \quad [\because D = b^2 - 4ac]$

Now, solution is: $x = \dfrac{20 \pm \sqrt{1156}}{2 \times 3} \quad \left[\because x = \dfrac{-b \pm \sqrt{D}}{2a}\right]$

$\Rightarrow \quad x = \dfrac{20 \pm 34}{6}; \Rightarrow x = \dfrac{20+34}{6}$ or $x = \dfrac{20-34}{6}$

$\Rightarrow \quad x = \dfrac{54}{6}$; or $x = \dfrac{-14}{6}$

$\Rightarrow \quad x = 9$; or $x = \dfrac{-7}{3}$ (Rejected)

$\therefore$ The required numbers are 9 and 13.

Q8.

Let the speed of the stream = km/h

Speed of the boat in still water = 20 km/h

Now, speed of boat during downstream = (20 + x) km/h

Distance covered during downstream = 48km.

$$\text{Time taken} = \left(\frac{48}{20+x}\right) \text{hours.}$$

Speed of boat during upstream = $(20-x)$ km/h

Distance covered during upstream = 48 km.

$$\text{Time taken} = \left(\frac{48}{20-x}\right) \text{hours.}$$

According to question, $\frac{48}{20-x} = 1 + \frac{48}{20+x}$

$$\frac{48}{20-x} - \frac{48}{20+x} = 1$$

$$\Rightarrow \quad \frac{960+48x-960+48x}{400-x^2} = 1$$

$$\Rightarrow \quad 96x = 400-x^2$$

$$\Rightarrow \quad x^2 + 96x - 400 = 0. \text{ As we know, } x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-96 \pm \sqrt{(96)^2+4\times1\times400}}{2\times1} \Rightarrow x = \frac{-96 \pm \sqrt{10816}}{2}$$

$$\Rightarrow \quad x = \frac{-96 \pm 104}{2}$$

$$\Rightarrow \quad x = \frac{-96+104}{2} \text{ or } x = \frac{-96-104}{2}$$

$$\Rightarrow \quad x = \frac{8}{2} = 4 \text{ or } x = \frac{-200}{2} = -100 \text{ (Not possible)}$$

Thus, speed of stream = 4km/h

Q9.

Simplifying given quadratic equation,

$$mx(x-7)+49 = 0, \text{ We get}$$

$$\Rightarrow \quad mx^2-7mx+49 = 0$$

In above equation, $a = m, b = -7m, c = 49$

For equal roots, $D = 0$, where D is discriminant

$$D = b^2-4ac$$

$$0 = (-7m)^2-4\times m\times 49$$

$$0 = 49m^2-196m$$

$$\Rightarrow \quad 49m^2-196m = 0 \Rightarrow 7m(7m-28)=0$$

$$\Rightarrow \quad 7m = 0 \text{ or } 7m-28=0 \Rightarrow m=0 \text{ or } 7m=28$$

$$\Rightarrow \quad m = 0 \text{ or } m=\frac{28}{7}=4$$

But $m \neq 0$ [$\because$ In quadratic equation, $a \neq 0$]

$$\therefore \quad m = 4$$

Q10.

Case (*i*) Let no. of students = x
Cost of food for each member = ₹ y
Total cost = ₹ 2000

$$x \times y = 2{,}000 \quad \ldots(1)$$

Case (*ii*) New no. of student = $x - 5$
New cost of food for each member = ₹ $(y + 20)$
Total cost = ₹ 2,000

$$(x - 5)(y + 20) = 2{,}000$$
$$xy + 20x - 5y - 100 = 2{,}000 \quad \ldots(2)$$

Solving (1) and (2), we get

$$2000 + 20x - 5y - 100 = 2{,}000$$
$$20x - 5y = 100$$
$$4x - y = 20$$
$$4x - \frac{2000}{x} = 20 \quad \text{[From (i)]}$$
$$x - \frac{500}{x} = 5$$
$$\Rightarrow \quad x^2 - 5x - 500 = 0 \Rightarrow x^2 - 25x + 20x - 500 = 0$$
$$\Rightarrow \quad x(x - 25) + 20(x - 25) = 0 \Rightarrow (x + 20)(x - 25) = 0$$
$$\Rightarrow \quad x + 20 = 0 \text{ or } x - 25 = 0$$
$$\therefore \quad x = -20, 25$$

$x = -20$ is rejected because no. of students can't be negative.

So, $x = 25$

$$y = \frac{2000}{x} = \frac{2000}{25} = 80$$

$\therefore \quad y = 80$

No. of students = 25
Cost of food for each student = ₹ 80.