Chapter-Quadratic Equations
Question bank

Q1.

).If the roots of the quadratic equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ in x are equal then show that either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$

Q2.

.If the roots of the quadratic equation

$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ are equal. Then show that $a = b = c$.

Q3.

.If the equation $(1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2(1+m^2)$

Q4.

If (-5) is a root of the quadratic equation $2x^2 + px + 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the values of p and k.

Q5.

.Write all the values of p for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained.

Q6.

.Find x in terms of a, b and c :

$$\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}, x \neq a, b, c$$

Q7.

.Solve for x : $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=2$ where $x \neq -\frac{1}{2}, 1$

Q8.

2.If $ad \neq bc$, then prove that the equation $(a^2+b^2)x^2+2(ac+bd)x+(c^2+d^2)=0$ has no real roots.

Q9.

Two water taps together can fill a tank in 6 hours. The tap of larger diameter takes 9 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Q10.

Prove that if a quadratic equation has rational coefficients and an irrational root of the form $p+\sqrt{q}$, then its other root must be $p-\sqrt{q}$.

Solutions

Solution 1.

We have $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$A = (c^2 - ab), B = (a^2 - bc), C = (b^2 - ac)$

If roots are equal, $B^2 - 4AC = 0$

$$[2(a^2 - bc)]^2 - 4(c^2 - ab)(b^2 - ac) = 0$$

$$4[a^4 + b^2c^2 - 2a^2bc] - 4(b^2c^2 - c^3a - ab^3 - a^2bc) = 0$$

$$4[a^4 + b^2c^2 - 2a^2bc - b^2c^2 + c^3a + ab^3 - a^2bc] = 0$$

$$4[a^4 + ac^3 + ab^3 - 3a^2bc = 0$$

$$a(a^3 + c^3 + b^3 - 3abc = 0$$

$$a = 0 \text{ or } a^3 + b^3 + c^3 = 3abc$$

Solution 2.

$$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$$

$$x^2-ax-bx+ab+$$

$$+x^2-bx-cx+bc+$$

$$+x^2-cx-ax+ac=0$$

$$3x^2-2ac-2bx-2cx+ab+bc+ca=0$$

For equal roots $B^2-4AC=0$

$$\{-2(a+b+c)\}^2-4\times 3(ab+bc+ca)=0$$

$$4(a+b+c)^2-12(ab+bc+ca)=0$$

$$a^2+b^2+c^2-3(ab+bc+ca)=0$$

$$a^2+b^2+c^2+2ab+2bc+2ac-3ab-3bc-3ac=0$$

$$a^2+b^2+c^2-ab-ac-bc=0$$

$$\tfrac{1}{2}[2a^2+2b^2+2c^2-2ab-2ac-2bc]=0$$

$$\tfrac{1}{2}[(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ac)]=0$$

$$\tfrac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]=0$$

or, $$(a-b)^2+(b-c)^2+(c-a)^2=0$$

If $a\neq b\neq c$

$$(a-b)^2>0,\ (b-c)^2>0,\ (c-a)^2>0$$

If $(a-b)^2=0\Rightarrow a=b$

$(a-c)^2=0\Rightarrow b=c$

$(c-a)^2=0\Rightarrow c=a$

Thus $a=b=c$ Hence Proved

Solution 3.

We have $(1+m^2)x^2+2mcx+(c^2-a^2)=0$

Comparing with $Ax^2+Bx+C=0$ we get

$$A=1+m^2, B=2mc, C=(c^2-a^2)$$

If roots are equal, $B^2-4AC=0$

$$(2mc)^2-4(1+m^2)(c^2-a^2)=0$$

$$4m^2c^2-4(1+m^2)(c^2-a^2)=0$$

$$m^2c^2-(c^2-a^2+m^2c^2-m^2a^2=0$$

$$m^2c^2-c^2+a^2-m^2c^2+m^2a^2=0$$

$$-c^2+a^2+m^2a^2=0$$

, $$c^2=a^2(1+m^2)$$

Hence Proved.

Solution 4.

We have $\quad 2x^2 + px - 15 = 0$

Since $x = -5$ is the root of above equation. It must satisfy it.

$$\begin{aligned} 2(-5)^2 + p(-5) - 15 &= 0 \\ 50 - 5p - 15 &= 0 \\ 5p &= 35 \Rightarrow p = 7 \end{aligned}$$

Now $\quad p(x^2 + x) + k = 0$ has equal roots

or $\quad 7x^2 + 7x + k = 0$

Taking $b^2 - 4ac = 0$ we have

$$\begin{aligned} 7^2 - 4 \times 7 \times k &= 0 \\ 7 - 4k &= 0 \\ k &= \frac{7}{4} \end{aligned}$$

Hence $p = 7$ and $k = \frac{7}{4}$.

Solution 5.

We have $x^2 + px + 16 = 0$ ...(1)

If this equation has equal roots, then discriminant $b^2 - 4ac$ must be zero.

i.e., $b^2 - 4ac = 0$ ...(2)

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 1$, $b = p$ and $c = 16$

Substituting above in equation (2) we have

$$p^2 - 4 \times 1 \times 16 = 0$$

$$p^2 = 64 \Rightarrow p = \pm 8$$

When $p = 8$, from equation (1) we have

$$x^2 + 8x + 16 = 0$$

$$x^2 + 2 \times 4x + 4^2 = 0$$

$$(x+4)^2 = 0 \Rightarrow x = -4, -4$$

Hence, roots are -4 and -4.

When $p = -8$ from equation (1) we have

$$x^2 - 8x + 16 = 0$$

$$x^2 - 2 \times 4x + 4^2 = 0$$

$$(x-4)^2 = 0 \Rightarrow x = 4, 4$$

Hence, the required roots are either $-4, -4$ or $4, 4$

Solution 6.

We have $\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}$

$$a(x-b)(x-c)+b(x-a)(x-c)=2c(x-a)(x-b)$$

$$ax^2-abx-acx+abc+bx^2-bax-bcx+abc$$

$$=2cx^2-2cxb-2cxa+2abc$$

$$ax^2+bx^2-2cx^2-abx-acx-bax-bcx+2cbx+2acx$$

$$=0$$

$$x^2(a+b-2c)-2abx+acx+bcx=0$$

$$x^2(a+b-2c)+x(-2ab+ac+bc)=0$$

Thus $x=-\left(\frac{ac+bc-2ab}{a+b-2c}\right)$

Solution 7.

We have $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=2$

Let $\frac{x-1}{2x+1}$ be y so $\frac{2x+1}{x-1}=\frac{1}{y}$

Substituting this value we obtain

$$y+\frac{1}{y}=2$$

$$y^2+1=2y$$

$$y^2-2y+1=0$$

$$(y-1)^2=0$$

$$y=1$$

Substituting $y=\frac{x-1}{2x+1}$ we have

$$\frac{x-1}{2x+1}=1 \text{ or } x-1=2x+1$$

or $$x=-2$$

Solution 8.

We have $(a^2+b^2)x^2+2(ac+bd)x+(c^2+d^2)=0$

Comparing with $Ax^2+Bx+C=0$ we get

$A=(a^2+b^2), B=2(ac+bd)$ and $C=c^2+d^2$

For no real roots, $D=B^2-4AC<0$

$$\begin{aligned}
D &= B^2-4AC \\
&= [2(ac+bd)]^2-4(a^2+b^2)(c^2+d^2) \\
&= 4[a^2c^2+2abcd+b^2d^2]-4[a^2c^2+a^2d^2+b^2c^2+b^2d^2] \\
&= 4[a^2c^2+2abcd+b^2d^2-a^2c^2-a^2d^2-b^2c^2-b^2d^2] \\
&= -4[a^2d^2+b^2c^2-2abcd] \\
&= -4(ad-bc)^2
\end{aligned}$$

Since $ad \neq bc$, therefore $D \neq 0$ and always negative.

Hence the equation has no real roots.

Solution 9.

Let x hours be the time taken by smaller one water tap. $\therefore$ $(x-9)$ hours be the time taken by larger one water tap.

According to question,

$$\frac{1}{x}+\frac{1}{x-9}=\frac{1}{6} \Rightarrow \frac{x-9+x}{x(x-9)}=\frac{1}{6}$$

$\Rightarrow$ $6(2x-9)=x(x-9)$

$\Rightarrow$ $12x-54=x^2-9x$

$\Rightarrow$ $x^2-21x+54=0$

$\Rightarrow$ $x^2-18x-3x+54=0 \Rightarrow x(x-18)-3(x-18)=0$

$\Rightarrow$ $(x-3)(x-18)=0$

$\Rightarrow$ $x=3$ or $x=18$

$\therefore$ Tap of smaller diameter takes 18 hours and that of larger diameter takes $(18-9)=9$ hrs.

Solution 10.

We need to **prove that if a quadratic equation has rational coefficients and an irrational root of the form** $p + \sqrt{q}$**, then its other root must be** $p - \sqrt{q}$.

Let the given quadratic equation be:

$$ax^2 + bx + c = 0$$

where a, b, c are rational numbers.

Step 1: Define the Roots

Given that one root is $p + \sqrt{q}$, let the other root be r.
Since the sum and product of the roots of a quadratic equation are given by:

$$\text{Sum of roots} = -\frac{b}{a}$$

$$\text{Product of roots} = \frac{c}{a}$$

Step 2: Check for Rational Coefficients

Since the coefficients of the quadratic equation are rational, the sum and product of the roots must also be rational.

If the second root r were not $p - \sqrt{q}$, the sum:

$$(p + \sqrt{q}) + r$$

would contain $\sqrt{q}$, making it irrational.

For the sum to be rational, the irrational term $\sqrt{q}$ must cancel out. This is only possible if $r = p - \sqrt{q}$, as then:

$$(p + \sqrt{q}) + (p - \sqrt{q}) = 2p$$

which is rational.

Similarly, the product:

$$(p + \sqrt{q})(p - \sqrt{q}) = p^2 - q$$

is also rational, as required.