CHAPTER- Probability

Introduction to Probability

Probability is the measure of the likelihood of an event occurring. It is expressed as a number between 0 and 1:

- 0 means the event is impossible.
- 1 means the event is certain.

The classical definition of probability is:

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

Key Concepts

1. Experiment

An experiment is any activity or process that leads to a result or outcome. For example:

- Tossing a coin
- Rolling a die
- Drawing a card from a deck

2. Sample Space

The sample space is the set of all possible outcomes of an experiment. For example:

- Tossing a coin: Sample space = {Head, Tail}
- Rolling a die: Sample space = {1, 2, 3, 4, 5, 6}

3. Event

An event is a subset of the sample space. It includes one or more outcomes of an experiment. For example:

- Rolling an even number on a die: Event = {2, 4, 6}

4. Complementary Events

If E is an event, its complement E' includes all outcomes not in E.

$$P(E') = 1 - P(E)$$

Properties of Probability

1. The probability of any event lies between 0 and 1:
2. $0 \leq P(E) \leq 1$
3. The sum of probabilities of all elementary events in an experiment is always 1.
4. The probability of an impossible event is 0.
5. The probability of a sure (certain) event is 1.

Solved Examples

Example 1: Tossing a Coin

A coin is tossed once. Find the probability of getting:

a) A head

b) A tail

Solution:

Sample space = {Head, Tail}

Total outcomes = 2

a) Probability of getting a head:

$$P(\text{Head}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{1}{2}$$

b) Probability of getting a tail:

$$P(\text{Tail}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{1}{2}$$

Example 2: Rolling a Die

A die is rolled once. Find the probability of getting:

a) A number greater than 4

b) An even number

Solution:

Sample space = {1, 2, 3, 4, 5, 6}

Total outcomes = 6

a) Probability of getting a number greater than 4 (favorable outcomes = {5, 6}):

$$P(\text{Greater than 4}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{2}{6} = \frac{1}{3}$$

b) Probability of getting an even number (favorable outcomes = {2, 4, 6}):

$$P(\text{Even number}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{3}{6} = \frac{1}{2}$$

Example 3: Drawing Cards from a Deck

A card is drawn from a standard deck (52 cards). Find the probability that the card is:

a) A king

b) Not a king

Solution:

Total cards in the deck = 52

a) Probability of drawing a king (favorable outcomes = {4 kings}):

$$P(\text{King}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{4}{52} = \frac{1}{13}$$

b) Probability of not drawing a king:

$$P(\text{Not King}) = P(E') = 1 - P(\text{King}) = 1 - \frac{1}{13} = \frac{12}{13}$$

Example 4: Rolling Two Dice Simultaneously

Two dice are rolled simultaneously. Find the probability that the sum is equal to 7.

Solution:

Sample space for two dice has $6 \times 6 = 36$ total outcomes.

Favorable outcomes for sum equal to 7: {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)} →

Total favorable outcomes = 6

Probability that the sum equals 7:

$$P(\text{Sum equals 7}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{6}{36} = \frac{1}{6}$$

Example 5: Complementary Events

In a bag containing 5 red balls, 3 blue balls, and 2 green balls, find the probability that:

a) A ball drawn is green

b) A ball drawn is not green

Solution:

Total balls in the bag = $5 + 3 + 2 = 10$

a) Probability that the ball drawn is green (favorable outcomes for green balls = {2}):

$$P(\text{Green}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{2}{10} = \frac{1}{5}$$

b) Probability that the ball drawn is not green:

$$P(\text{Not Green}) = P(E') = 1 - P(\text{Green}) = 1 - \frac{1}{5} = \frac{4}{5}$$
