Chapter- Polynomials

Introduction to Polynomials

A polynomial is an algebraic expression consisting of variables and coefficients, involving only addition, subtraction, multiplication, and non-negative integer exponents.
General form: $a_0x^n + a_1x^{n-1} + ... + a_{n-1}x + a_n$
Example 1:
Identify the polynomial: $3x^2 - 2x + 5$
Solution: This is a quadratic polynomial (degree 2)
Example 2:
Is $2/x + 3$ a polynomial?
Solution: No, because it involves division by a variable

Zeros of a Polynomial

A zero of a polynomial p(x) is a value of x for which p(x) = 0.
Example 1:
Find the zeros of $p(x) = x^2 - 1$
Solution:
$x^2 - 1 = 0$
$(x+1)(x-1) = 0$
$x = 1$ or $x = -1$
Example 2:
Find the zero of $p(x) = 2x + 3$
Solution:
$2x + 3 = 0$
$2x = -3$
$x = -3/2$

Relationship between Zeros and Coefficients of Quadratic Polynomials

For a quadratic polynomial $ax^2 + bx + c$:

- Sum of zeros = $-b/a$
- Product of zeros = c/a

Example 1:
If the zeros of $x^2 - 5x + 6$ are α and β, find $\alpha + \beta$ and $\alpha\beta$.

Solution:

a = 1, b = -5, c = 6
α + β = -b/a = -(-5)/1 = 5
αβ = c/a = 6/1 = 6

Example 2:

The sum of zeros of a quadratic polynomial is 7 and their product is 12. Find the polynomial.
Solution:
Let the polynomial be $x^2 + bx + c$
Sum of zeros = -b/1 = 7, so b = -7
Product of zeros = c/1 = 12, so c = 12
The polynomial is $x^2 - 7x + 12$