Chapter-Pair of Linear Equations in Two Variables

1. Introduction to Linear Equations in Two Variables

A linear equation in two variables is of the form $ax + by + c = 0$, where a, b, and c are real numbers and a and b are not both zero.

Example 1:
Identify if the following is a linear equation in two variables: $2x + 3y = 7$
Solution: Yes, it is a linear equation in two variables x and y.

Example 2:
Is $x^2 + y = 5$ a linear equation in two variables?
Solution: No, it's not linear due to the x^2 term.

2. Pair of Linear Equations

A pair of linear equations in two variables is a system of two equations:
$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$

Example 1:
Write the following as a pair of linear equations:
The sum of a two-digit number and the number formed by reversing its digits is 121. The difference between the two numbers is 9.
Solution:
Let the two-digit number be $10x + y$
Reversed number: $10y + x$
Equation 1: $(10x + y) + (10y + x) = 121$
Equation 2: $(10x + y) - (10y + x) = 9$

Example 2:
Represent the following situation as a pair of linear equations:
The cost of 3 pens and 2 notebooks is ₹80, while 2 pens and 3 notebooks cost ₹90.
Solution:
Let x be the cost of a pen and y be the cost of a notebook
Equation 1: $3x + 2y = 80$
Equation 2: $2x + 3y = 90$

3. Graphical Method of Solution

The solution of a pair of linear equations is the point of intersection of their graphs.

Example 1:
Solve graphically: $x + y = 5$ and $x - y = 1$

Solution:
Plot both lines on a graph. They intersect at (3, 2), which is the solution.
Example 2:
Solve graphically: $2x + y = 8$ and $x + y = 5$
Solution:
Plot both lines. They intersect at (3, 2), which is the solution.

4. Algebraic Methods of Solution

a. Substitution Method

Example 1:
Solve using substitution: $x + y = 5$ and $x - y = 1$
Solution:
From equation 1: $y = 5 - x$
Substitute in equation 2: $x - (5 - x) = 1$
$2x - 5 = 1$
$2x = 6$
$x = 3$
$y = 5 - 3 = 2$
Solution: (3, 2)
Example 2:
Solve: $2x + 3y = 13$ and $x + y = 5$
Solution:
From equation 2: $x = 5 - y$
Substitute in equation 1: $2(5 - y) + 3y = 13$
$10 - 2y + 3y = 13$
$10 + y = 13$
$y = 3$
$x = 5 - 3 = 2$
Solution: (2, 3)

b. Elimination Method

Example 1:
Solve using elimination: $3x + 2y = 11$ and $2x - 3y = -5$
Solution:
Multiply equation 1 by 2 and equation 2 by 3:
$6x + 4y = 22$
$6x - 9y = -15$

Add these equations:
$13y = 37$
$y = 37/13$
Substitute y in equation 1:
$3x + 2(37/13) = 11$
$x = 1$
Solution: (1, 37/13)

Example 2:
Solve: $5x - 2y = 16$ and $3x + 2y = 14$
Solution:
Add the equations:
$8x = 30$
$x = 15/4$
Substitute x in equation 1:
$5(15/4) - 2y = 16$
$y = 1/4$
Solution: (15/4, 1/4)

5. Conditions for Consistency

A pair of linear equations can have:

- Exactly one solution (intersecting lines)
- Infinitely many solutions (coincident lines)
- No solution (parallel lines)

Example 1:
Determine the nature of the solution for: $2x + 3y = 7$ and $4x + 6y = 14$
Solution:
Comparing coefficients: $2/4 = 3/6 = 7/14$
The equations are equivalent, so there are infinitely many solutions.

Example 2:
Determine the nature of the solution for: $2x + 3y = 7$ and $2x + 3y = 8$
Solution:
The equations are inconsistent ($2/2 = 3/3 \neq 7/8$), so there is no solution.