Chapter-Pair of Linear Equations in Two Variables
Question bank

Q1.

On reversing the digits of a two digit number, number obtained is 9 less than three times the original number. If difference of these two numbers is 45, find the original number.

Q2.

4. Find the values of p and q for which the following pair of linear equations has infinitely many solutions:
$(p+q)x + 3y = p-q$
$6x + (p-q)y = 3p+q$

Q3.

5. The sum of the digits of a two-digit number is 9. If 27 is added to the number, the digits are reversed. Find the number.

Q4.

8. Two numbers are in the ratio 5:3. If 9 is subtracted from the larger number and 3 is added to the smaller number, the ratio becomes 2:1. Find the numbers.

Q5.

9. A motorboat goes 25 km upstream and 39 km downstream in 6 hours. In 6½ hours, it can go 27 km upstream and 42 km downstream. Find the speed of the motorboat in still water and the speed of the stream.

Q6.

Solve the following pair of linear equations for x and y:
$141x + 93y = 189;$
$93x + 141y = 45$ (2013)

Q7.

On reversing the digits of a two digit number, number obtained is 9 less than three times the original number. If difference of these two numbers is 45, find the original number. (´

Q8.

The owner of a taxi company decides to run all the taxis on CNG fuel instead of petrol/diesel. The taxi charges in city comprises of fixed charges together with the charge for the distance covered. For a journey of 12 km, the charge paid is 789 and for journey of 20 km, the charge paid is ₹145.
What will a person have to pay for travelling a distance of 30 km?

Q9.

Find the value of a and p for which the following pair of linear equations has infinite number of solutions:
$2x + 3y = 7$;
$\alpha x + (\alpha + \beta)y = 28$ (2013)

Q10.

A boat takes 4 hours to go 44 km downstream and it can go 20 km upstream in the same time. Find the speed of the stream and that of the boat in still water. (2015)

Solutions

Q1.

Let unit's place digit be x and ten's place digit bey.

∴ Original number = x + 10y Reversed number = 10x + y

According to the Question,

10x + y = 3(x + 10y) − 9

⇒ 10x + y = 3x + 30y − 9

⇒ 10x + y − 3x − 30y = -9

⇒ 7x − 29y = -9 ...(i)

10x + y − (x + 10y) = 45

⇒ 9x − 9y = 45

⇒ x − y = 5 ...[Dividing both sides by 9

⇒ x − 5 + y ...(ii)

Solving (i),

7x − 29y = -9

7(5 + y) − 29y = -9 ...[From (ii)

35+ 7y − 29y = -9

-22y = -9 − 35

-22y = -44 ⇒ $y = \frac{44}{22} = 2$

Putting the value of y in (ii),

x = 5 + 2 = 7

∴ Original number = x + 10y

= 7 + 10(2) = 27

Q2.

Solution:

Step 1: For infinitely many solutions, the ratios of coefficients must be equal:

$(p+q)/6 = 3/(p-q) = (p-q)/(3p+q)$

Step 2: From the first equality:

$(p+q)(p-q) = 18$

$p^2 - q^2 = 18$

Step 3: From the second equality:

$3(3p+q) = (p-q)(p-q)$

$9p + 3q = p^2 - 2pq + q^2$

Step 4: Substitute $p^2 = 18 + q^2$ from step 2 into the equation from step 3:

$9p + 3q = 18 + q^2 - 2pq + q^2$

$9p + 3q = 18 + 2q^2 - 2pq$

Step 5: Solve this equation along with $p^2 - q^2 = 18$ to get:

$p = 3$ and $q = -3$

Therefore, $p = 3$ and $q = -3$.

Q3.

Solution:

Step 1: Let the tens digit be x and the units digit be y.

Step 2: Set up the equations:

$x + y = 9$ (sum of digits)

$10x + y + 27 = 10y + x$ (reversing digits after adding 27)

Step 3: Simplify the second equation:

$9x - 9y = -27$

$x - y = -3$

Step 4: Solve the system of equations:

$x + y = 9$

$x - y = -3$

Adding these equations:

$2x = 6$

$x = 3$

Step 5: Substitute $x = 3$ in $x + y = 9$:

$3 + y = 9$

$y = 6$

Therefore, the number is 36.

Q4.

Solution:
Step 1: Let the larger number be 5x and the smaller number be 3x.
Step 2: Set up the equation:
(5x - 9) / (3x + 3) = 2/1
Step 3: Cross multiply:
1(5x - 9) = 2(3x + 3)
5x - 9 = 6x + 6
Step 4: Solve for x:
-x = 15
x = -15
Step 5: Calculate the numbers:
Larger number = 5(-15) = -75
Smaller number = 3(-15) = -45
However, these negative numbers don't make sense in the context of the problem.
Step 6: Let's try again with the larger number as 3x and the smaller number as 5x:
(3x - 9) / (5x + 3) = 2/1
1(3x - 9) = 2(5x + 3)
3x - 9 = 10x + 6
-7x = 15
x = -15/7
Step 7: Calculate the numbers:
Larger number = 3(-15/7) = -45/7
Smaller number = 5(-15/7) = -75/7
These are still negative, so our initial assumption about the ratio was incorrect.
Step 8: Let's try one more time with the larger number as 5x and the smaller number as 3x:
(5x - 9) / (3x + 3) = 2/1
5x - 9 = 6x + 6
-x = 15
x = -15
This gives us the same result as step 5, which doesn't work.

Therefore, this problem has no solution with positive real numbers.

Q5.

Solution:
Step 1: Let the speed of the motorboat in still water be x km/h and the speed of the stream be y km/h.
Step 2: Set up the equations:
25/(x-y) + 39/(x+y) = 6
27/(x-y) + 42/(x+y) = 6.5
Step 3: Multiply both equations by (x^2 - y^2):
25(x+y) + 39(x-y) = 6(x^2 - y^2)
27(x+y) + 42(x-y) = 6.5(x^2 - y^2)
Step 4: Simplify:
64x - 14y = 6x^2 - 6y^2
69x - 15y = 6.5x^2 - 6.5y^2
Step 5: Subtract the first equation from the second:
5x - y = 0.5x^2 - 0.5y^2
10x - 2y = x^2 - y^2
x^2 - 10x - y^2 + 2y = 0
(x^2 - 10x + 25) - (y^2 - 2y + 1) = 26
(x - 5)^2 - (y - 1)^2 = 26
Step 6: This is a difference of squares. Let x - 5 = u and y - 1 = v:
u^2 - v^2 = 26
(u+v)(u-v) = 26
The only integer factors of 26 that work are 13 and 2.
u + v = 13 and u - v = 2
2u = 15
u = 7.5
Step 7: Solve for x and y:
x - 5 = 7.5
x = 12.5
y - 1 = 5.5
y = 6.5
Therefore, the speed of the motorboat in still water is 12.5 km/h and the speed of the stream is 6.5 km/h.

Q6.

$$\begin{array}{rrrl} 141x & + & 93y & = 189 \\ 93x & + & 141y & = 45 \\ \hline 234x & + & 234y & = 234 \end{array}$$ ...[By adding

$\Rightarrow \quad x + y = 1$ ...(*i*) (+ by 234)

Again

$$\begin{array}{rrrl} 141x & + & 93y & = 189 \\ 93x & + & 141y & = 45 \\ \hline 48x & - & 48y & = 144 \end{array}$$ ...[By subtracting

$\Rightarrow \quad x - y = 3$ ...(*ii*) (+ by 48)

By adding (*i*) and (*ii*), we get

$$\begin{array}{rl} x + y = 1 & \ldots(i) \\ x - y = 3 & \ldots(ii) \\ \hline 2x = 4 & \Rightarrow \quad x = 2 \end{array}$$

Putting the value of x in (*i*), we get

$2 + y = 1$

$y = 1 - 2 = -1 \quad \therefore \quad x = 2, y = -1$

Q7.

Let unit's place digit be x and ten's place digit bey.

∴ Original number = x + 10y Reversed number = 10x + y

According to the Question,

10x + y = 3(x + 10y) − 9

⇒ 10x + y = 3x + 30y − 9

⇒ 10x + y − 3x − 30y = -9

⇒ 7x − 29y = -9 ...(i)

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35+ 7y − 29y = -9

-22y = -9 − 35

-22y = -44 ⇒ $y = \frac{44}{22} = 2$

Putting the value of y in (ii),

x = 5 + 2 = 7

∴ Original number = x + 10y

Q8.

Let the fixed charges = 7x

and the charge per km = ₹y

According to the Question,

$x + 12y = 89$ ...(i)

$x + 20y = 145$ ...(ii)

$-8y = -56$ ...[By subtracting

$y = \frac{-56}{-8} = 7$

Putting the value of y in (i), we get

x + 12(7) = 89

x + 84 = 89 ⇒ x = 89 − 84 = 5

Total fare for 30 km = x + 30y = 5 + 30(7)

= 5 + 210 = ₹215

Q9.

We have, $2x + 3y = 7$ and $\alpha x + (\alpha + \beta)y = 28$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ...[For Infinite many solutions

$\frac{2}{\alpha} = \frac{3}{\alpha + \beta} = \frac{7^1}{28^4}$

$\frac{2}{\alpha} = \frac{1}{4}$

$\alpha = 8$ ...(i)

$\frac{3}{\alpha + \beta} = \frac{1}{4}$

$\alpha + \beta = 12$

$8 + \beta = 12$ ...[From (i)

$\beta = 12 - 8 = 4$

$\therefore \alpha = 8, \beta = 4$

Q10.

Let the speed of the stream = y km/hr

Let the speed of boat in still water = x km/hr

then, the speed of the boat in downstream = (x + y) km/hr

and, the speed of the boat in upstream = (x – y) km/hr

According to Question,

$\frac{44}{x + y} = 4$

$4(x + y) = 44$ 4

$\Rightarrow x + y = 11$

$\Rightarrow x = 11 - y$ ...(i)

$\frac{20}{x - y} = 4$

...[$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$(x - y) = 20$

$x - y = 5$

$11 - y - y = 5$ ...[From (i)

$2y = 6$

$\Rightarrow y = 3$

From (i), x = 11 – 3 = 8

∴ Speed of the stream, y =3 km/hr

Speed of the boat in still water, x = 8 km/hr