Chapter-Introduction to Trigonometry

1. Trigonometric Ratios of an Acute Angle of a Right-Angled Triangle

In a right-angled triangle ABC, right-angled at B:

- sin A = Opposite side / Hypotenuse = BC / AC
- cos A = Adjacent side / Hypotenuse = AB / AC
- tan A = Opposite side / Adjacent side = BC / AB
- cot A = Adjacent side / Opposite side = AB / BC
- sec A = Hypotenuse / Adjacent side = AC / AB
- cosec A = Hypotenuse / Opposite side = AC / BC

Example 1:

In a right-angled triangle, if sin θ = 3/5, find the values of cos θ and tan θ.

Solution:

Using Pythagoras theorem: $\sin^2 \theta + \cos^2 \theta = 1$

$(3/5)^2 + \cos^2 \theta = 1$

$9/25 + \cos^2 \theta = 1$

$\cos^2 \theta = 1 - 9/25 = 16/25$

$\cos \theta = 4/5$

$\tan \theta = \sin \theta / \cos \theta = (3/5) / (4/5) = 3/4$

Example 2:

If cos A = 5/13, find the values of other trigonometric ratios.

Solution:

In a right-angled triangle, let the adjacent side be 5 and hypotenuse be 13.

Using Pythagoras theorem: $5^2 + x^2 = 13^2$

$25 + x^2 = 169$

$x^2 = 144$

x = 12 (opposite side)

Now we can calculate:

sin A = 12/13

tan A = 12/5

cot A = 5/12

sec A = 13/5

cosec A = 13/12

2. Trigonometric Ratios of Some Specific Angles (0°, 30°, 45°, 60°, 90°)

Angle	sin	cos	tan
0°	0	1	0
30°	1/2	√3/2	1/√3
45°	1/√2	1/√2	1
60°	√3/2	1/2	√3
90°	1	0	∞

Example 1:

Find the value of sin 60° + cos 30°.

Solution:

sin 60° = √3/2

cos 30° = √3/2

sin 60° + cos 30° = √3/2 + √3/2 = √3

Example 2:

Evaluate: tan 45° × cot 45°

Solution:

tan 45° = 1

cot 45° = 1

tan 45° × cot 45° = 1 × 1 = 1

3. Trigonometric Identities

1. $\sin^2 A + \cos^2 A = 1$

Example 1:
Prove that $(1 - \sin A)(1 + \sin A) = \cos^2 A$
Solution:
LHS = $(1 - \sin A)(1 + \sin A)$
= $1 - \sin^2 A$
= $\cos^2 A$ (using identity 1)
= RHS
Example 2:
If $\sin A = 3/5$, find the value of $\cos A + \tan A$.
Solution:
Using $\sin^2 A + \cos^2 A = 1$:
$(3/5)^2 + \cos^2 A = 1$
$\cos^2 A = 1 - 9/25 = 16/25$
$\cos A = 4/5$
$\tan A = \sin A / \cos A = (3/5) / (4/5) = 3/4$
$\cos A + \tan A = 4/5 + 3/4 = 16/20 + 15/20 = 31/20$