Chapter-Introduction to Trigonometry
Question bank

Q1.

. Prove that:

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\mathrm{cosec}\,\theta$$

Q2.

.Prove that $b^2x^2-a^2y^2=a^2b^2$, if :

(1) $x=a\sec\theta, y=b\tan\theta$, or

(2) $x=a\,\mathrm{cosec}\,\theta$, $y=b\cot\theta$

Q3.

.Prove that :

$$\frac{\cot^3\theta\sin^3\theta}{(\cos\theta+\sin\theta)^2}+\frac{\tan^3\theta\cos^3\theta}{(\cos\theta+\sin\theta)^2}=\frac{\sec\theta\,\mathrm{cosec}\,\theta-1}{\mathrm{cosec}\,\theta+\sec\theta}$$

Q4.

.If $\cos\theta+\sin\theta=p$ and $\sec\theta+\mathrm{cosec}\,\theta=q$, prove that $q(p^2-1)=2p$

Q5.

.If $x=r\sin A\cos C$, $y=r\sin A\sin C$ and $z=r\cos A$, then prove that $x^2+y^2+z^2=r^2$

Q6.

If cosθ-sinθ=√2 sinθ,prove that cosθ+sinθ=√2 cosθ

Q7.

If cosec (A-B) = 2, cot (A + B) = —0° < (A + B) <90°, A > B, then find A and B

Q8.

If 3x= cosec θ and 3/x= cot θ, find the value of $3(x^2-1/x^2)$

Q9.

Find the value of cosec 30° geometrically.

Q10.

Show that $\tan^4\theta+\tan^2\theta=\sec^4\theta-\sec^2\theta$

Solutions

Q1.

L.H.S. =

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$=\frac{1}{\sin\theta-\cos\theta}\left[\frac{\sin^2\theta}{\cos\theta}-\frac{\cos^2\theta}{\sin\theta}\right]$$

$$=\frac{1}{\sin\theta-\cos\theta}\left[\frac{\sin^3\theta-\cos^3\theta}{\cos\theta\cdot\sin\theta}\right]$$

$$=\frac{[\sin\theta-\cos\theta][\sin^2\theta+\cos^2\theta+\sin\theta\cdot\cos\theta]}{(\sin\theta-\cos\theta)\cdot(\cos\theta\cdot\sin\theta)}$$

$$=\frac{\sin^2\theta+\cos^2\theta+\sin\theta\cdot\cos\theta}{(\cos\theta\cdot\sin\theta)}$$

$$=\frac{1+\sin\theta\cdot\cos\theta}{\cos\theta\cdot\sin\theta}\qquad[\because \sin^2\theta+\cos^2\theta=1]$$

$$=\frac{1}{\cos\theta\cdot\sin\theta}+\frac{\sin\theta\cdot\cos\theta}{\sin\theta\cdot\cos\theta}$$

$= 1 + \sec\theta. \operatorname{cosec}\theta$ [R.H.S.]

Hence Proved.

Q2.

(1) We have $x = a\sec\theta, y = b\tan\theta,$

$$\frac{x^2}{a^2} = \sec^2\theta,\ \frac{y^2}{b^2} = \tan^2\theta$$

or, $$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2\theta - \tan^2\theta = 1$$

Thus $\quad b^2x^2 - a^2y^2 = a^2b^2 \qquad$ Hence Proved

(ii) We have $x = a\,\text{cosec}\,\theta, y = b\cot\theta$

$$\frac{x^2}{a^2} = \text{cosec}^2\theta,\ \frac{y^2}{b^2} = \cot^2\theta$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \text{cosec}^2\theta - \cot\theta = 1$$

Thus $\quad b^2x^2 - a^2y^2 = a^2b^2 \qquad$ Hence Proved

Q3.

$$\frac{\cot^3\theta\sin^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\tan^3\theta\cos^3\theta}{(\cos\theta+\sin\theta)^2}$$

$$= \frac{\frac{\cos^3\theta}{\sin^3\theta}\times\sin^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\frac{\sin^3\theta}{\cos^3\theta}\times\cos^3\theta}{(\cos\theta+\sin\theta\ ^2}$$

$$= \frac{\cos^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\sin^3\theta}{(\cos\theta+\sin\theta)^2}$$

$$= \frac{(\cos\theta+\sin\theta)(\cos^2\theta+\sin^2\theta-\sin\theta\cos\theta)}{(\cos\theta+\sin\theta)^2}$$

$$= \frac{1-\sin\theta\cos\theta}{\cos\theta+\sin\theta} = \frac{\frac{1}{\cos\theta\sin\theta} - \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}}{\frac{\cos\theta}{\cos\theta\sin\theta} + \frac{\sin\theta}{\cos\theta\sin\theta}}$$

$$= \frac{\text{cosec}\,\theta\sec\theta - 1}{\text{cosec}\,\theta + \sec\theta} \qquad \text{Hence Proved}$$

Q4.

We have $\cos\theta + \sin\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$

$$q(p^2 - 1) = (\sec\theta + \operatorname{cosec}\theta)[(\cos\theta + \sin\theta)^2 - 1]$$
$$= (\sec\theta + \operatorname{cosec}\theta)(\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 1)$$
$$= (\sec\theta + \operatorname{cosec}\theta)[1 + 2\sin\theta\cos\theta - 1]$$
$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta)$$
$$= \left(\frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}\right)2\sin\theta\cos\theta$$
$$= 2(\sin\theta + \cos\theta) = 2p$$

Hence Proved.

Q5.

Since, $x^2 = r^2\sin^2 A\cos^2 C$

$y^2 = r^2\sin^2 A\sin^2 C$

and $z^2 = r^2\cos^2 A$

$$x^2 + y^2 + z^2 = r^2\sin^2 A\cos^2 C + r^2\sin^2 A\sin^2 C + r^2\cos^2 A$$
$$= r^2\sin^2 A(\cos^2 C + \sin^2 C) + r^2\cos^2 A$$
$$= r^2\sin^2 A + r^2\cos^2 A$$
$$= r^2(\sin^2 A + \cos^2 A)$$
$$= r^2$$

Hence Proved.

Q6.

Here, $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

$\Rightarrow$ $\cos\theta = \sqrt{2}\sin\theta + \sin\theta \Rightarrow \cos\theta = (\sqrt{2}+1)\sin\theta$

$\Rightarrow$ $\dfrac{\cos\theta}{\sqrt{2}+1} = \sin\theta \Rightarrow \dfrac{\cos\theta(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} = \sin\theta$

$\Rightarrow$ $\sqrt{2}\cos\theta - \cos\theta = \sin\theta \Rightarrow \sqrt{2}\cos\theta = \sin\theta + \cos\theta$

Hence proved.

Q7.

$$\text{cosec}(A - B) = 2$$

Also, $\text{cosec}\, 30^\circ = 2 \Rightarrow A - B = 30^\circ$ ...(*i*)

and $\cot(A + B) = \frac{1}{\sqrt{3}}$

Also, $\cot 60^\circ = \frac{1}{\sqrt{3}} \Rightarrow A + B = 60^\circ$ ...(*ii*)

Solving (*i*) and (*ii*), we get $A = 45^\circ$, $B = 15^\circ$

Q8.

$3x = \text{cosec}\,\theta$ and $\frac{3}{x} = \cot\theta$

$$x = \frac{\text{cosec}\,\theta}{3} \text{ and } \frac{1}{x} = \frac{\cot\theta}{3}$$

$$3\left[x^2 - \frac{1}{x^2}\right] = 3\left[\left(\frac{\text{cosec}\,\theta}{3}\right)^2 - \left(\frac{\cot\theta}{3}\right)^2\right] = 3\left[\frac{\text{cosec}^2\theta - \cot^2\theta}{9}\right] = 3 \times \frac{1}{9} = \frac{1}{3}$$

Q9.

For the value of cosec 30° consider an equilateral triangle ABC.

Let $2a$ be the length of each side

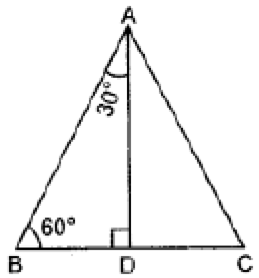
$\Rightarrow$ $AB = BC = CA = 2a$

Since each angle of equilateral triangle is 60°.

$\therefore$ $\Delta ABD \cong \Delta ACD$

$\therefore$ $BD = DC$ (CPCT)

and $\angle BAD = \angle CAD$



ΔABD is right angled triangle.

$$BD = \frac{1}{2} BC = a$$

Now, $\text{cosec}\, 30^\circ = \frac{AB}{BD} = \frac{2a}{a} = 2$

Hence, the value of cosec 30° = 2.

Q10.

Consider,

$$\text{L.H.S.} = \tan^4\theta + \tan^2\theta$$
$$= \tan^2\theta(\tan^2\theta + 1)$$
$$= (\sec^2\theta - 1)\sec^2\theta$$
$$[\because \sec^2\theta - \tan^2\theta = 1]$$
$$= \sec^4\theta - \sec^2\theta = \text{R.H.S.}$$

Hence Proved.