Chapter-Introduction to Trigonometry
Question bank

Q1.

If $\sec\theta = x + \frac{1}{4x}, x \neq 0$, find $(\sec\theta + \tan\theta)$.

Q2.

Q. 5. Prove that

$$\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\operatorname{cosec}^2 A}{\sec^2 A - \operatorname{cosec}^2 A} = \frac{1}{1 - 2\cos^2 A}$$

Q3.

Prove that:

$$\frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} = 2 + \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$$

Q4.

In an acute angled triangle ABC if $\sin(A+B-C) = \frac{1}{2}$ and $\cos(B+C-A) = \frac{1}{\sqrt{2}}$ find $\angle A, \angle B$ and $\angle C$.

Q5.

.If $7\operatorname{cosec}\phi - 3\cot\phi = 7$, prove that $7\cot\phi - 3\operatorname{cosec}\phi = 3$.

Q6,

.If $\tan A + \sin A = m$ and $\tan A - \sin A = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

Q7.

.Prove that $\sec^2\theta + \operatorname{cosec}^2\theta$ can never be less than 2.

Q8.

..If $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$
$= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$
Prove that each of the side is equal to ± 1.

Q9

2.If $4\sin\theta = 3$, find the value of x if

$$\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} + 2\cot\theta = \frac{\sqrt{7}}{x} + \cos\theta$$

Q10

If Sin θ +Sin^2 θ +Sin^3 θ = 1 , then find the value of Cos^6 θ -4 Cos^4 θ +8 Cos^2 θ.

Solution

Solution 1.

Given,

$$\sec\theta = x + \frac{1}{4x} \qquad \text{...(i)}$$

Squaring both sides, we get

$$\sec^2\theta = \left(x + \frac{1}{4x}\right)^2$$

$$= x^2 + \frac{1}{16x^2} + 2 \times x \times \frac{1}{4x}$$

$$= x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

Also, $\sec^2\theta = 1 + \tan^2\theta$

$$\therefore \quad 1 + \tan^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

or $$\tan^2\theta = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$= \left(x - \frac{1}{4x}\right)^2$$

$$\Rightarrow \quad \tan\theta = \left(x - \frac{1}{4x}\right) \text{ or } -\left(\frac{1}{4x} - x\right) \text{...(ii)}$$

Now, from equation (i) and (ii)

$$\sec\theta + \tan\theta = x + \frac{1}{4x} + x - \frac{1}{4x}$$

$$= 2x$$

or $$\sec\theta + \tan\theta = x + \frac{1}{4x} + \frac{1}{4x} - x$$

$$= \frac{1}{2x}.$$

Hence,

$$\sec\theta + \tan\theta = 2x \text{ or } \frac{1}{2x}$$

Q2.

Ans. $$\text{L.H.S.} = \frac{\tan^2 A}{\tan^2 A - 1} + \frac{\operatorname{cosec}^2 A}{\sec^2 A - \operatorname{cosec}^2 A}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A}} + \frac{\frac{1}{\sin^2 A}}{\frac{1}{\cos^2 A} - \frac{1}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\frac{1}{\sin^2 A}}{\frac{\sin^2 A - \cos^2 A}{\sin^2 A \cdot \cos^2 A}}$$

$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\cos^2 A}{\sin^2 A - \cos^2 A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin^2 A - \cos^2 A} = \frac{1}{\sin^2 A - \cos^2 A}$$

$$= \frac{1}{1 - \cos^2 A - \cos^2 A} \quad [\because \sin^2 A = 1 - \cos^2 A]$$

$$= \frac{1}{1 - 2\cos^2 A}$$

$= \text{R.H.S.}$ **Hence Proved.**

Q3.

L.H.S.

$$\frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} = \frac{\sin\theta}{\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta+1}}{\sin\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta+1}$$

$$= \frac{\sin^2\theta}{1+\cos\theta} \times \frac{(1-\cos\theta)}{(1-\cos\theta)}$$

$$= \frac{\sin^2\theta(1-\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{\sin^2\theta(1-\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{\sin^2\theta(1-\cos\theta)}{\sin^2\theta}$$

$$= 1 - \cos\theta \qquad \text{...(i)}$$

R.H.S.

$$2 + \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta} = 2 + \frac{\sin\theta}{\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}}$$

$$= 2 + \frac{\sin^2\theta}{\cos\theta - 1}$$

$$= 2 - \frac{\sin^2\theta}{(1-\cos\theta)}$$

$$= 2 - \frac{\sin^2\theta \times (1+\cos\theta)}{(1-\cos\theta)\times(1+\cos\theta)}$$

$$= 2 - \frac{\sin^2\theta(1+\cos\theta)}{1-\cos^2\theta}$$

$$= 2 - \frac{\sin^2\theta\,(1+\cos\theta)}{\sin^2\theta}$$

$$= 2 - (1 + \cos\theta)$$

$$= 1 - \cos\theta \qquad \text{...(ii)}$$

From equation (i) and (ii), we get

L.H.S. = R.H.S. **Hence Proved.**

Q4.

We have $\sin(A+B-C) = \frac{1}{2} = \sin 30^o$

$$A+B-C = 30^o \qquad ...(1)$$

and $\cos(B+C-A) = \frac{1}{\sqrt{2}} = \cos 45^o$

$$B+C-A = 45^o \qquad ...(2)$$

Adding equation (1) and (2), we get

$$2B = 75^o \Rightarrow B = 37.5^o$$

Subtracting equation (2) from equation (1) we get,

$$2(A-C) = -15^o$$

$$A-C = -7.5^o \qquad ...(3)$$

Now $A+B+C = 180^o$

$$A+C = 180^{\circ} - 37.5^{\circ} = 142.5^o \qquad ...(4)$$

Adding equation (3) and (4), we have

$$2A = 135^o \Rightarrow A = 67.5^o$$

and, $C = 75^o$

Hence, $\angle A = 67.5^o, \angle B = 37.5^o, \angle C = 75^o$

Q5.

$$
\begin{aligned}
\text{We have}\quad 7\operatorname{cosec}\phi - 3\cot\phi &= 7 \\
7\operatorname{cosec}\phi - 7 &= 3\cot\phi \\
7(\operatorname{cosec}\phi - 1) &= 3\cot\phi \\
7(\operatorname{cosec}\phi - 1)(\operatorname{cosec}\phi + 1) &= 3\cot\phi(\operatorname{cosec}\phi + 1) \\
7(\operatorname{cosec}^2\phi - 1) &= 3\cot\phi(\operatorname{cosec}\phi + 1) \\
7\cot^2\phi &= 3\cot\phi(\operatorname{cosec}\phi + 1) \\
7\cot\phi &= 3\operatorname{cosec}\phi + 3 \\
7\cot\phi - 3\operatorname{cosec}\phi &= 3
\end{aligned}
$$

Hence proved

Q6,

We have $\tan A + \sin A = m$

and $\tan A - \sin A = n$

$$m^2 - n^2 = (\tan A + \sin A)^2 - (\tan A - \sin A)^2$$

$$= (\tan^2 A + \sin^2 A + 2\sin A \tan A)$$

$$-(\tan^2 A + \sin^2 A - 2\sin A \tan A)$$

$$= \tan^2 A + \sin^2 A + 2\sin A \tan A$$

$$-\tan^2 A - \sin^2 A + 2\sin A \tan A$$

$$= 4\sin A \tan A$$

$$4\sqrt{mn} = 4\sqrt{(\tan A + \sin A)(\tan A - \sin A)}$$

$$= 4\sqrt{\tan^2 A - \sin^2 A}$$

$$= 4\sqrt{\frac{\sin^2 A}{\cos^2 A} - \sin^2 A}$$

$$= 4\sqrt{\frac{\sin^2 A - \sin^2 A \cos^2 A}{\cos^2 A}}$$

$$= 4\sqrt{\frac{\sin^2 A(1 - \cos^2 A)}{\cos^2 A}}$$

$$= 4\sqrt{\frac{\sin^2 A \times \sin^2 A}{\cos^2 A}}$$

$$= 4\frac{\sin A \times \sin A}{\cos A}$$

$$= 4\sin A \times \frac{\sin A}{\cos A}$$

$$= 4\sin A \tan A$$

Thus $m^2 - n^2 = 4\sqrt{mn}$ Hence Proved

Q7.

Let $\sec^2\theta + \operatorname{cosec}^2\theta = x$

$$1 + \tan^2\theta + 1 + \cot^2\theta = x$$

$$2 + \tan^2\theta + \cot^2\theta = x$$

$$2 + \tan^2\theta + \cot^2\theta = x$$

$\tan^2\theta \geq 0$ and $\cot^2\theta \geq 0$

Thus $x > 2$

Thus $\sec^2\theta + \operatorname{cosec}^2\theta > 2$

Hence $\sec^2\theta + \operatorname{cosec}^2\theta$ can never be less than 2.

Q8.

Ans :

We have

$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$ h207

$= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

Multiply both sides by

$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

or, $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) \times$

$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

$= (\sec A - \tan A)^2(\sec B - \tan B)^2(\sec C - \tan C)^2$

or, $(\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$

$= (\sec A - \tan A)^2(\sec A - \tan B)^2(\sec C - \tan C)^2$

or, $1 = \left[(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)\right]^2$

or, $(\sec A - \tan A)(\sec B - \tan B)(\sec C + \tan C) = \pm 1$

Q9

We have $\sin\theta = \frac{3}{4}$

or, $\sin^2\theta = \frac{9}{16}$

Since $\sin^2\theta + \cos^2 = 1$, we have

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{9}{16} = \frac{7}{16}$$

$$\cos\theta = \frac{\sqrt{7}}{4}$$

and $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{4}}{\frac{\sqrt{7}}{4}} = \frac{3}{\sqrt{7}}$

Thus $\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} + 2\cot\theta = \frac{\sqrt{7}}{x} + \cos\theta$

$$\sqrt{\frac{1}{\tan^2\theta}} + 2 \times \frac{\sqrt{7}}{3} = \frac{\sqrt{7}}{x} + \frac{\sqrt{7}}{4}$$

$$\frac{1}{\tan\theta} + \frac{2\sqrt{7}}{3} = \frac{\sqrt{7}}{x} + \frac{\sqrt{7}}{4}$$

$$\frac{\sqrt{7}}{3} + \frac{2\sqrt{7}}{3} - \frac{\sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$

$$\frac{4\sqrt{7} - \sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$

$$\frac{3\sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$

Thus $x = \frac{4}{3}$

Q10

Given $\sin\theta + \sin^2\theta + \sin^3\theta = 1$

$\implies \sin\theta(1 + \sin^2\theta) = 1 - \sin^2\theta$

$\implies \sin\theta(2 - \cos^2\theta) = \cos^2\theta$

Squaring on both sides

$\implies \sin^2\theta(2 - \cos^2\theta)^2 = \cos^4\theta$

$\implies (1 - \cos^2\theta)(4 + \cos^4\theta - 4\cos^2\theta) = \cos^4\theta$

$\implies 4 + \cos^4\theta - 4\cos^2\theta - 4\cos^2\theta - \cos^6\theta + 4\cos^4\theta = \cos^4\theta$

$\implies \cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$