Chapter-Introduction to Trigonometry
Question bank

Q1.

If (1 + cos A) (1 – cos A) =3/4, find the value of sec A.

Q2.

Given tan A = 5/12 , find the other trigonometric ratios of the angle A.

Q3.

If sin (A + B) = 1 and tan (A – B) = 1/√3, find the value of:

1. tan A + cot B
2. sec A – cosec B

Q4.

If cosec θ = 5/3, then what is the value of cos θ + tanθ

Q5.

Evaluate: 3 $\cot^2 60°$ + $\sec^2 45°$.

Q6.

$$\text{Evaluate}: \frac{5\cos^2 60^\circ + 4\cos^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 60^\circ}$$

Q7.

$$\text{Verify}: \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta}, \text{ for } \theta = 60^\circ$$

Q8.

$$\text{Prove that}: (\cot\theta - \operatorname{cosec}\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$$

Q9.

Prove that : $\dfrac{\cos A}{1+\tan A}-\dfrac{\sin A}{1+\cot A}=\cos A-\sin A$

Q10.

In ΔABC, $\angle B=90^{\circ}$, $BC=5$ cm, $AC-AB=1$,
Evaluate : $\dfrac{1+\sin C}{1+\cos C}$.

Solution 1.

$$(1 + \cos A)(1 - \cos A) = \frac{3}{4}$$

$$\therefore \quad 1 - \cos^2 A = \frac{3}{4}$$

$$1 - \frac{3}{4} = \cos^2 A$$

$$\frac{1}{4} = \cos^2 A \Rightarrow \sec^2 A = 4 \Rightarrow \sec A = \pm 2$$

Solution 2.

Given: $\tan A = \frac{5}{12}$

As $\cot A = \frac{1}{\tan A} = \frac{12}{5}$

$$\text{cosec}^2 A = 1 + \cot^2 A = 1 + \left(\frac{12}{5}\right)^2 = 1 + \frac{144}{25} = \frac{169}{25}$$

$$\text{cosec}^2 A = \frac{169}{25}$$

$\Rightarrow$ $\text{cosec } A = \frac{13}{5}$

Now, $\text{cosec } A = \frac{1}{\sin A} \Rightarrow \frac{13}{5} = \frac{1}{\sin A}$

$$\text{Sin } A = \frac{5}{13}$$

Also, $\cos^2 A = 1 - \sin^2 A = 1 - \frac{25}{169} = \frac{169 - 25}{169} = \frac{144}{169}$

$$\cos^2 A = \frac{144}{169}$$

$\therefore$ $\cos A = \frac{12}{13}$

As $\cos A = \frac{1}{\sec A}$

$\therefore$ $\sec A = \frac{13}{12}$

Solution 3.

$$\sin(A + B) = 1 \quad \text{(Given)}$$
$$\Rightarrow \quad \sin(A + B) = \sin 90^\circ \quad \text{(As } \sin 90^\circ = 1\text{)}$$
$$\Rightarrow \quad A + B = 90^\circ \quad \ldots(i)$$

Also $$\tan(A - B) = \frac{1}{\sqrt{3}} \quad \text{(Given)}$$
$$\tan(A - B) = \tan 30^\circ \quad \left(\text{As } \tan 30^\circ = \frac{1}{\sqrt{3}}\right)$$
$$\Rightarrow \quad A - B = 30^\circ \quad \ldots(ii)$$

Solving (*i*) and (*ii*) for A and B, we get $A = 60^\circ$ and $B = 30^\circ$

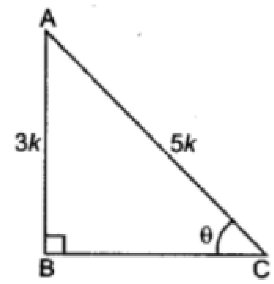
(*i*) $\tan A + \cot B = \tan 60^\circ + \cot 30^\circ = \sqrt{3} + \sqrt{3} = 2\sqrt{3}$

(*ii*) $\sec A - \operatorname{cosec} B = \sec 60^\circ - \operatorname{cosec} 30^\circ = 2 - 2 = 0$

Solution 4.

$$\because \quad \operatorname{cosec}\theta = \frac{5}{3}$$

In right angled ΔABC, $\angle B = 90^\circ$



So, $$AC^2 = AB^2 + BC^2 \quad \text{[By Pythagoras theorem]}$$
$$\Rightarrow \quad (5k)^2 = (3k)^2 + BC^2$$
$$\Rightarrow \quad BC^2 = 25k^2 - 9k^2$$
$$\Rightarrow \quad BC^2 = 16k^2 \Rightarrow BC = 4k$$

So, $$\cos\theta = \frac{4}{5} \text{ and } \tan\theta = \frac{3}{4}$$

Now, $$\cos\theta + \tan\theta = \frac{4}{5} + \frac{3}{4} = \frac{16 + 15}{20} = \frac{31}{20}$$

Solution 5.

$$3\cot^2 60^\circ + \sec^2 45^\circ = 3\left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{2})^2 = 3 \times \frac{1}{3} + 2 = 1 + 2 = 3$$

Solution 6.

$$\frac{5\cos^2 60^o + 4\cos^2 30^o - \tan^2 45^o}{\sin^2 30^o + \cos^2 60^o}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{\sqrt{3}}{2}\right)^2 \quad (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \frac{\frac{5}{4} + 3 - 1}{\frac{1}{4} + \frac{1}{4}}$$

$$= \frac{\frac{5}{4} + 2}{\frac{1}{2}} = \frac{\frac{13}{4}}{\frac{1}{2}} = \frac{13}{2}$$

Solution 7.

$$\text{LHS} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\cos 60^o}{1+\cos 60^o}}$$

$$= \sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}} = \sqrt{\frac{\frac{1}{2}}{\frac{3}{2}}} = \frac{1}{\sqrt{3}} \qquad \left(\cos 60^o = \frac{1}{2}\right)$$

$$\text{RHS} = \frac{\sin\theta}{1+\cos\theta} = \frac{\sin 60^o}{1+\cos 60^o}$$

$$= \frac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{1}{\sqrt{3}}$$

$$\text{RHS} = \text{LHS}$$

Hence, relation is verified for $\theta = 60^o$.

Solution 8.

$$\cot\theta - \operatorname{cosec}\theta = \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}$$

$$(\cot\theta - \operatorname{cosec}\theta)^2 = \left(\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)^2$$

$$= \left(\frac{\cos\theta - 1}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} \quad [[\sin^2\theta + \cos^2\theta = 1]]$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}$$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta} \qquad \text{Hence Proved.}$$

Solution 9.

$$\frac{\cos A}{1+\tan A} - \frac{\sin A}{1+\cot A}$$

$$= \frac{\cos A}{1+\frac{\sin A}{\cos A}} - \frac{\sin A}{1+\frac{\cos A}{\sin A}}$$

$$= \frac{\cos^2 A}{\cos A + \sin A} - \frac{\sin^2 A}{\sin A + \cos A}$$

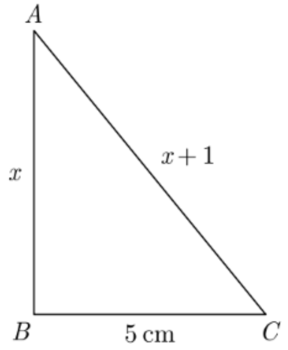
$$= \frac{\cos^2 A - \sin^2 A}{(\sin A + \cos A)}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\sin A + \cos A}$$

$$= \cos A - \sin A \qquad \text{Hence Proved.}$$

Solution 10.

As per question we have drawn the figure given below.



We have $\quad AC - AB = 1$

Let $AB = x$, then we have

$$AC = x + 1$$

Now
$$AC^2 = AB^2 + BC^2$$
$$(x+1)^2 = x^2 + 5^2$$
$$x^2 + 2x + 1 = x^2 + 25$$
$$2x = 24$$
$$x = \frac{24}{2} = 12 \text{ cm}$$

Hence, $AB = 12$ cm and $AC = 13$ cm

Now
$$\sin C = \frac{AB}{AC} = \frac{12}{13}$$
$$\cos C = \frac{BC}{AC} = \frac{5}{13}$$

Now
$$\frac{1 + \sin C}{1 + \cos C} = \frac{1 + \frac{12}{13}}{1 + \frac{5}{13}} = \frac{\frac{25}{13}}{\frac{18}{13}} = \frac{25}{18}$$