Chapter-Coordinate Geometry
Question bank

Q1.

If the point P(x, y) is equidistant from the points A (a + b, b – a) and B(a -b,a+ b). Prove that bx = ay.

Q2.

If the coordinates of points A and B are (-2, -2) and (2, – 4) respectively, find the coordinates of P such that AP = 3/7 AB, where P lies on the line segment AB.

Q3.

Point A lies on the line segment PQ joining P(6, -6) and Q(-4, -1) in such a way that PA/PQ=2/5. If point P also lies on the line 3x + k(y + 1) = 0, find the value of k

Q4.

Points A(-l, y) and B(5,7) lie on a circle with centre 0(2, -3y). Find the values ofy. Hence, find the radius of the circle.

Q5.

. **Find the ratio in which $P(4, m)$ divides the line segment joining the points $A(2, 3)$ and $B(6, -3)$. Hence find m. [CBSE, 2018]**

Q6. Find a point on the y-axis which is equidistant from the points A(-5, 2) and B(9, -2).

Q7. Find the coordinates of the points of trisection of the line segment joining (4, -1) and (-2, -3).

Q8. If A (-2, 4), B (0, 0) and C (4, 2) are the vertices of triangle ABC, then find the length of the median through the vertex A.

Q9. Show that the points (-3, 2), (-5, -5), (2, -3), and (4, 4) are the vertices of a rhombus. Find the area of this rhombus.

Q10.

Find a relation between x and y such that the point (x , y) is equidistant from the points (7, 1) and (3, 5).

Solutions

Solution 1.

Solution:

Given, $PA = PB \Rightarrow PA^2 = PB^2$

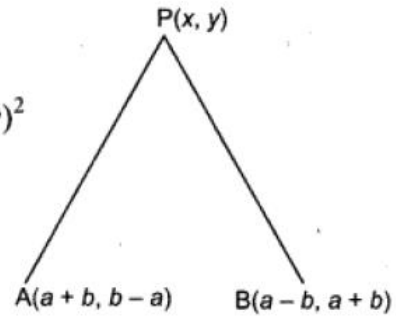
Applying distance formula,

$\Rightarrow (a + b - x)^2 + (b - a - y)^2 = (a - b - x)^2 + (a + b - y)^2$

$\Rightarrow (a + b)^2 + x^2 - 2ax - 2bx + (b - a)^2 + y^2 - 2by + 2ay$

$= (a - b)^2 + x^2 - 2ax + 2bx + (a + b)^2 + y^2 - 2ay - 2by$

$\Rightarrow 4ay = 4bx \Rightarrow ay = bx$ or $bx = ay$ Hence proved.



Solution 2.

Given, $AP = \frac{3}{7} AB \Rightarrow AP : PB = 3 : 4$

$\Rightarrow$ P divides AB in the ratio 3 : 4

Using section formula,

$\therefore$ Coordinates of point P are $= \left(\frac{3 \times 2 + 4 \times (-2)}{3+4}, \frac{3 \times (-4) + 4 \times (-2)}{3+4}\right) = \left(\frac{-2}{7}, \frac{-20}{7}\right)$

Solution 3.

Coordinates of P are (6, –6). Given that:

$\because$ P(6, –6) lies on the line. So,

$3x + k(y + 1) = 0$

$\Rightarrow 3 \times 6 + k(-6 + 1) = 0$

$\Rightarrow 18 - 5k = 0$

$\Rightarrow k = \frac{18}{5}.$

Solution 4.

Given, O is the centre of the circle and the points A and B lie on the circle.

So, $OA = OB\ (= r)$ [$\because$ radius of same circle]

$\Rightarrow OA^2 = OB^2$

Using distance formula,

$\Rightarrow (2+1)^2 + (-3y-y)^2 = (2-5)^2 + (-3y-7)^2$

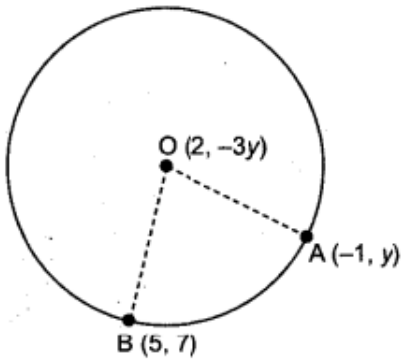
$\Rightarrow 9 + 16y^2 = 9 + 9y^2 + 42y + 49$

$\Rightarrow 7y^2 - 42y - 49 = 0$

$\Rightarrow y^2 - 6y - 7 = 0$

$\Rightarrow (y-7)(y+1) = 0$

$\Rightarrow y = -1$ or 7



When $y = -1$, then co-ordinates are: O(2, 3) and A(–1, –1)

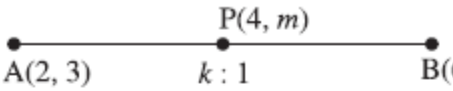
Radius of circle, $r = OA = \sqrt{(2+1)^2+(3+1)^2} = \sqrt{9+16} = 5$ units

When $y = 7$, then coordinates are: O(2, – 21) and A(–1, 7)

Radius of circle, $r = OA = \sqrt{(2+1)^2+(-21-7)^2} = \sqrt{9+784} = \sqrt{793}$ units.

Solution 5.

Let P divides line segment AB in the ratio $k : 1$.



$$P = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right)$$

$$(4, m) = \left(\frac{k \times 6 + 1 \times 2}{k+1}, \frac{k \times (-3) + 1 \times 3}{k+1} \right)$$

$$(4, m) = \left(\frac{6k+2}{k+1}, \frac{-3k+3}{k+1} \right) \quad \text{...(i)}$$

On comparing, we get

$$\left(\frac{6k+2}{k+1} \right) = 4$$

$$\Rightarrow \quad 6k + 2 = 4 + 4k$$

$$\Rightarrow \quad 6k - 4k = 4 - 2$$

$$\Rightarrow \quad 2k = 2$$

$$\Rightarrow \quad k = 1 \quad \text{...(ii)}$$

Hence, P divides AB in the ratio $1 : 1$.

From (i) and (ii), $\frac{-3(1)+3}{1+1} = m$

$$\Rightarrow \quad \frac{-3+3}{2} = m$$

$$\Rightarrow \quad m = 0$$

Solution 6:

Let P(0, y) be the required point on the y-axis.

Since P is equidistant from A(-5, 2) and B(9, -2),

PA = PB

$(0 - (-5))^2 + (y - 2)^2 = (0 - 9)^2 + (y - (-2))^2$

$25 + (y - 2)^2 = 81 + (y + 2)^2$

$25 + y^2 - 4y + 4 = 81 + y^2 + 4y + 4$

$25 + y^2 - 4y = 81 + y^2 + 4y$

$25 - 81 = 8y$

$-56 = 8y$

$y = -7$

Therefore, P(0, -7) is the required point

Solution 7.

Given coordinates are$(x_1, y_1) = (4, -1)$

and $(x_2, y_2) = (-2, -3)$

Means of trisection:- A line segment in three equal parts then ratio is 1 : 2 and 2 : 1 internally.

Case (1) If $m_1 : m_2 = 1 : 2$

Then, using formula

$$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$(x, y) = \left(\frac{1 \times (-2) + 2 \times 4}{1 + 2}, \frac{1 \times (-3) + 2 \times (-1)}{1 + 2}\right)$$

$$(x, y) = \left(\frac{-2 + 8}{3}, \frac{-3 - 2}{3}\right)$$

$$(x, y) = \left(2, -\frac{5}{3}\right)$$

Case (2):-

If $m_1 : m_2 = 2 : 1$

Then, using formula

$$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$(x, y) = \left(\frac{2 \times (-2) + 1 \times 4}{2 + 1}, \frac{2 \times (-3) + 1 \times (-1)}{1 + 2}\right)$$

$$(x, y) = \left(\frac{-4 + 4}{3}, \frac{-6 - 1}{3}\right)$$

$$(x, y) = \left(0, -\frac{7}{3}\right)$$

Solution 8:

The formula for the midpoint of a line segment connecting two points (x_1, y_1) and (x_2, y_2) is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

For points $B(0, 0)$ and $C(4, 2)$:

$$\text{Midpoint of } BC = \left(\frac{0+4}{2}, \frac{0+2}{2}\right) = (2, 1)$$

The distance formula between two points (x_1, y_1) and (x_2, y_2) is:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substituting $A(-2, 4)$ and the midpoint $(2, 1)$:

$$\text{Length of median} = \sqrt{(2 - (-2))^2 + (1 - 4)^2}$$

$$= \sqrt{(2+2)^2 + (-3)^2}$$

$$= \sqrt{4^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

Solution 9:

Let us consider the co-ordinates of the vertices A, B, C and D of a rhombus are A (-3, 2), B (-5, -5), C (2, -3) and D (4, 4)

$AB = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

$= \sqrt{[(-5 + 3)^2 + (-5 - 2)^2]} = \sqrt{[(-2)^2 + (-7)^2]}$

$\sqrt{[4 + 49]} = \sqrt{53}$

Similarly, $BC^2 = (2 + 5)^2 + (-3 + 5)^2$

$= (7)^2 + (2)^2 = 49 + 4 = 53$

$CD^2 = (4 - 2)^2 + (4 + 3)^2$

$= (2)^2 + (7)^2 = 4 + 48 = 53$

and $DA^2 = (-3 - 4)^2 + (2 - 4)^2$

$= (-7)^2 + (-2)^2 = 49 + 4 = 53$

Therefore, we see that AB = BC = CD = DA = $\sqrt{53}$

Now diagonals AC = $= \sqrt{(2+3)^2 + (-3-2)^2} = \sqrt{(5)^2 + (-5)^2} = \sqrt{25+25} = \sqrt{50} = \sqrt{25*2} = 5\sqrt{2}$

and diagonal BD = $= \sqrt{(4+5)^2 + (4+5)^2} = \sqrt{(9)^2 + (9)^2} = \sqrt{81+81} = \sqrt{162} = \sqrt{81*2} = 9\sqrt{2}$

Therefore, sides are equal but diagonal are not equal

Hence, ABCD is a rhombus

Now we find the area of rhombus = Product of diagonal/2

$= (5\sqrt{2} \times 9\sqrt{2})/2$

= 9/2

= 45sq.units

Solution 10: Let P(x, y) be equidistant from the points A(7, 1) and B(3, 5).
We are given that AP = BP. So, $AP^2 = BP^2$
i.e., $(x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$
i.e., $x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$
i.e., x - y = 2
which is the required relation.