Chapter- Circles

Introduction to Circles

A circle is a collection of all points in a plane that are at a constant distance from a fixed point called the center. The constant distance is called the radius.
Key terms:

- Center: The fixed point from which all points on the circle are equidistant
- Radius: The distance from the center to any point on the circle
- Chord: A line segment joining any two points on the circle
- Diameter: A chord passing through the center (longest chord)
- Tangent: A line that touches the circle at exactly one point

Tangents to a Circle

A tangent to a circle is a line that touches the circle at exactly one point, called the point of contact. The tangent is perpendicular to the radius drawn at the point of contact.

Example 1: If TP is a tangent to the circle at P and OP is the radius drawn to the point of contact, prove that $OP \perp TP$.
Solution: Assume OP is not perpendicular to TP.
Draw OQ perpendicular to TP.
OQ < OP (as perpendicular is the shortest distance from a point to a line).
But OQ cannot be shorter than OP as OP is a radius.
This contradiction proves that OP must be perpendicular to TP.

Example 2: Two tangents PA and PB are drawn to a circle with center O. If $\angle APB = 60°$, find $\angle AOP$.
Solution: In triangle OPA, $\angle OPA = 90°$ (tangent $\perp$ radius).
In triangle OPB, $\angle OPB = 90°$.
$\angle APB = 60°$,
so $\angle AOP + \angle BOP = 180° - 60° = 120°$.
As OA = OB (radii), $\angle OAP = \angle OBP$.
Therefore, $\angle AOP = \angle BOP = 120°/2 = 60°$.

Number of Tangents from a Point to a Circle

- From a point on the circle: Only one tangent can be drawn
- From a point inside the circle: No tangent can be drawn

- From a point outside the circle: Two tangents can be drawn

Example 1: A tangent PQ at a point P of a circle of radius 5 cm meets a line through the center O at a point Q so that OQ = 12 cm. Find the length PQ.

Solution: In right triangle OPQ, $OP \perp PQ$ (tangent $\perp$ radius).

OP = 5 cm (radius), OQ = 12 cm (given).

Using Pythagorean theorem in triangle OPQ:

$OQ^2 = OP^2 + PQ^2$

$12^2 = 5^2 + PQ^2$

$144 = 25 + PQ^2$

$PQ^2 = 119$

$PQ = \sqrt{119} \approx 10.91$ cm

Lengths of Tangents from an External Point

The lengths of tangents drawn from an external point to a circle are equal.

Example 1: Prove that the lengths of tangents drawn from an external point to a circle are equal.

Solution: Let PA and PB be two tangents from external point P to circle with center O.

Join OA, OB, and OP.

In triangles OPA and OPB:

OA = OB (radii),

OP = OP (common),

$\angle OAP = \angle OBP = 90^\circ$ (tangent $\perp$ radius).

By RHS congruence,

$\triangle OPA \cong \triangle OPB$.

Therefore, PA = PB (corresponding sides of congruent triangles).

Example 2: Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Solution: Let AB be the chord of the larger circle touching the smaller circle at point P. $OP \perp AB$ (tangent $\perp$ radius).

In the right triangle OPA: OP = 3 cm, OA = 5 cm.

Using Pythagorean theorem:

$AP^2 = OA^2 - OP^2$

$AP^2 = 5^2 - 3^2 = 25 - 9 = 16$

AP = 4 cm

AB = 2AP = 2 × 4 = 8 cm