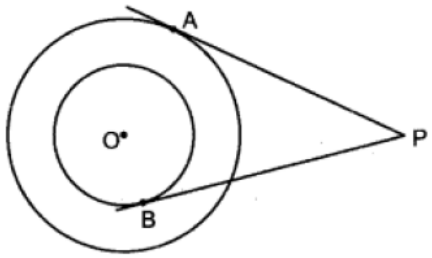
Chapter- Circles

Q1.

In figure, there are two concentric circles, with centre O and of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP.

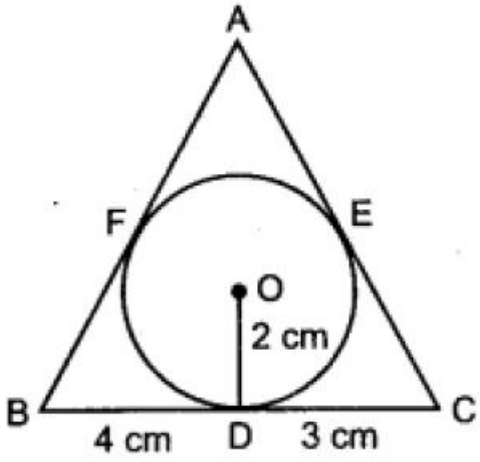


Q2.

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

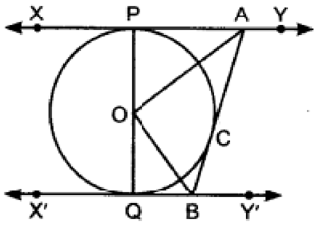
Q3.

In figure, a triangle ABC is drawn to circumscribe a circle of radius 2 cm such that the segments BD and DC into which BC is divided by the point of contact D are the lengths 4 cm and 3 cm respectively. If area of $\Delta ABC = 21\ cm^2$, then find the lengths of sides AB and AC.



Q4.

In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects XY at A and X'Y' at B. Prove that ∠AOB = 90°.



Q5.

8.In a right angle $\Delta\, ABC$, $BC = 12$ cm and $AB = 5$ cm.
Find the radius of the circle inscribed in this triangle.

Q6.

In figure 2, PQ is a chord of a circle with centre O and PT is a tangent. If $\angle QPT = 60°$, find $\angle PRQ$.

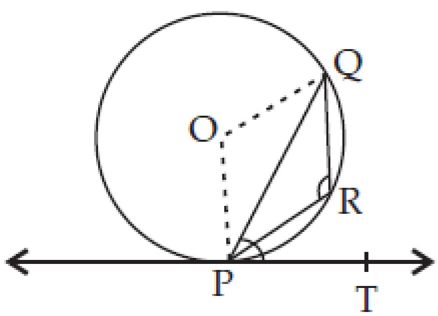
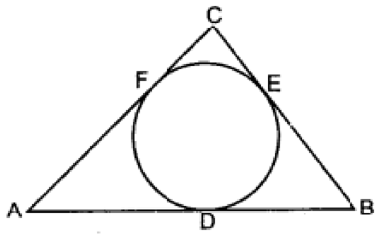


Figure 2

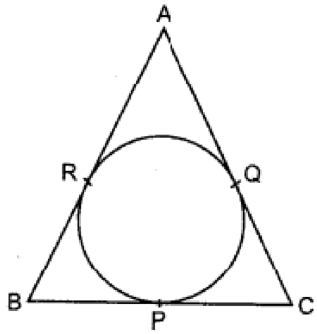
Q7.

In the given figure, a circle inscribed in ΔABC touches its sides AB, BC and AC at points D, E and F respectively. If AB = 12 cm, BC = 8 cm and AC = 10 cm, then find the lengths of AD, BE and CF



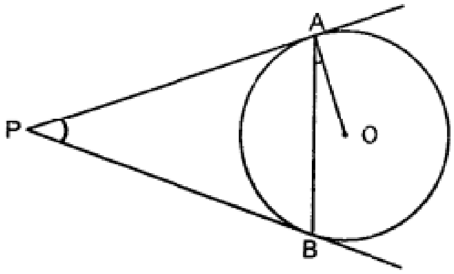
Q8.

In the given figure, a circle inscribed in ΔABC, touches its sides BC, CA and AB at the points P, Q and R respectively. If AB = AC, then prove that BP = CP.



Q9.

In the given figure, two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that ∠APB = 2 ∠OAB



Q10:

In Fig. , tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30°$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$.

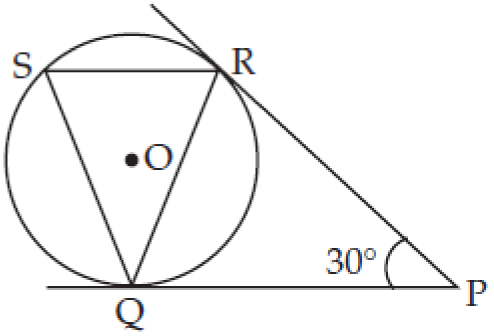


Figure ,

Solutions

Q1.

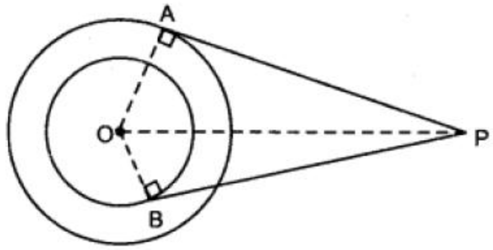
Solution:

Construction: Join OA, OB and OP.

AP = 12 cm, OA = 5 cm, OB = 3 cm

In ΔAOP, $\angle A = 90°$ [radius is perpendicular to the tangent at point of contact]

ΔBOP, $\angle B = 90°$ [radius is perpendicular to the tangent at point of contact]



So, $OP^2 = OA^2 + AP^2$

and $OP^2 = OB^2 + BP^2$

Using Pythagoras theorem for ΔAOP and ΔBOP.

$\therefore \quad OA^2 + AP^2 = OB^2 + BP^2$

$5^2 + 12^2 = 3^2 + BP^2 \Rightarrow 25 + 144 = 9 + BP^2 \Rightarrow 169 - 9 = BP^2$

$\Rightarrow \quad BP = \sqrt{160}$ cm = 12.65 cm

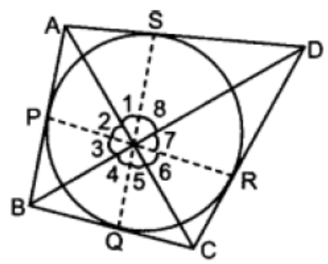
Q2.

Solution:

The given quadrilateral ABCD is circumscribing the circle having its centre at O.

The sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively.

Join OA, OB, OC, OD; OP, OQ, OR, OS.



We observe that OA bisects $\angle POS$ [$\because$ By CPCT, applied to ΔPOA and ΔSOA]

$\Rightarrow \quad \angle 1 = \angle 2$ ...(*i*)

similarly $\quad \angle 3 = \angle 4$ ...(*ii*)

$\angle 5 = \angle 6$ ...(*iii*)

and $\quad \angle 7 = \angle 8$ ...(*iv*)

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$ [$\because$ ASP of quadrilateral]

$\Rightarrow \quad 2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360°$

$\Rightarrow \quad (\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180° \Rightarrow \angle AOD + \angle BOC = 180°$

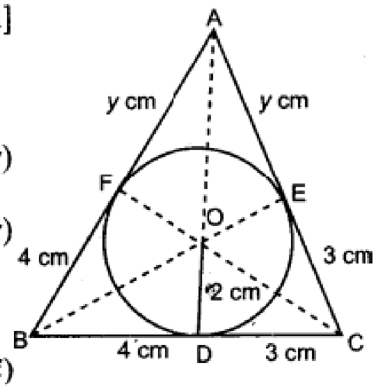
Similarly, $\quad \angle AOB + \angle COD = 180°$

Hence, opposite sides of the quadrilateral ABCD subtend supplementary angles at the centre

Q3.

Let AE = AF = y (say)

[Tangents drawn from an external point are equal]



$$\text{ar } \Delta BOC = \frac{1}{2} \times 7 \times 2 = 7 \text{ cm}^2 = b \text{ (say)}$$

$$\text{ar } \Delta AOB = \frac{1}{2} \times (4+y) \times 2 = (4+y) \text{ cm}^2 = a \text{ (say)}$$

$$\text{ar } \Delta AOC = \frac{1}{2} \times (3+y) \times 2 = (3+y) \text{ cm}^2 = c \text{ (say)}$$

Now, $\text{ar } \Delta ABC = \text{ar } \Delta AOB + \text{ar } \Delta BOC + \text{ar } \Delta AOC$

$= 4 + y + 7 + 3 + y$

$\text{ar } \Delta ABC = 14 + 2y$ ...(*i*)

For ΔABC,

Semi-perimeter, $$s = \frac{a+b+c}{2} = \frac{4+y+7+3+y}{2} = \frac{14+2y}{2} = 7 + y$$

$\therefore$

$$\begin{aligned} \text{ar } \Delta ABC &= \sqrt{s(s-a)(s-b)(s-c)} \quad [\because \text{ By Heron's formula}] \\ &= \sqrt{(7+y)(7+y-4-y)[7+y-7)(7+y-3-y)} \\ &= \sqrt{(7+y) \times 3 \times y \times 4} \\ \text{ar } \Delta ABC &= 2\sqrt{3y(7+y)} \quad ...(ii) \end{aligned}$$

From (*i*) and (*ii*)

$$\Rightarrow \quad 2\sqrt{3y(7+y)} = 14 + 2y$$

$$\Rightarrow \quad \sqrt{3y(7+y)} = 7 + y$$

Squaring both sides, we get

$$\Rightarrow \quad 3y(7+y) = (7+y)^2 \Rightarrow 21y + 3y^2 = 49 + y^2 + 14y$$

$$\Rightarrow \quad 2y^2 + 7y - 49 = 0 \Rightarrow 2y^2 + 14y - 7y - 49 = 0$$

$$\Rightarrow \quad 2y(y+7) - 7(y+7) = 0 \Rightarrow (2y-7)(y+7) = 0$$

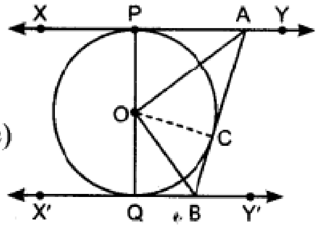
$$\Rightarrow \quad y = \frac{7}{2}, y = -7 \quad \text{[Rejected]}$$

Hence, length of side AB = 4 + 3.5 = 7.5 cm and AC = 3 + 3.5 = 6.5 cm.

Q4.

Solution:

Given: XY and X'Y' are are two parallel tangents to circle with centre O. Tangent AB with point of contact C intersects XY at A and X'Y' at B.



To Prove: $\angle AOB = 90°$

Construction: Join OC.

Proof: In ΔOPA and ΔOCA,

$OP = OC$ (Radii of the same circle)

$AP = AC$ (Tangents from point A)

$AO = AO$ (common side)

$\Delta OPA \cong \Delta OCA$ (SSS congruence rule)

Therefore, $P \rightarrow e, A \rightarrow A, O \rightarrow o$,

$\angle POA = \angle COA$ ...(*i*) (CPCT)

Similarly, we prove: $\Delta OQB \cong \Delta OCB$

Then: $\angle QOB = \angle COB$ ...(*ii*) (CPCT)

Since, POQ is the diameter of the circle, it is a straight line.

$\therefore \angle POA + \angle COA + \angle COB + \angle QOB = 180°$

from equation (*i*) and (*ii*),

$$2\angle COA + 2\angle COB = 180°$$

$$2(\angle COA + \angle COB) = 180°$$

$$\angle COA + \angle COB = \frac{180°}{2}$$

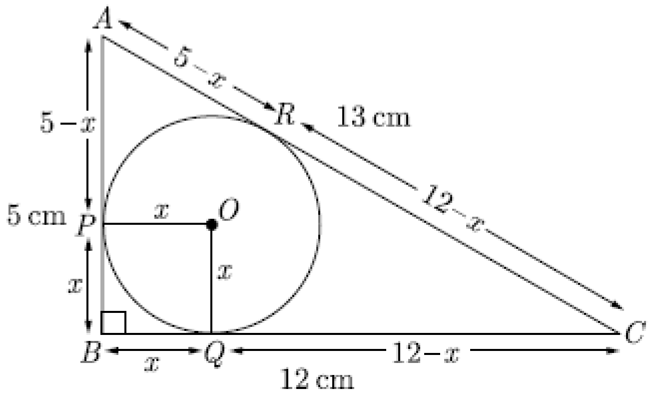
$$\angle COA + \angle COB = 90°$$

$$\angle AOB = 90°$$

Hence, Proved.

Q5.

Let the radius of circle be x. As per given in question we draw the figure shown below.



Since length of tangents from an external point to a circle are equal,

At A, $\quad AP = AR = 5 - x \quad (1)$

At B $\quad BP = BQ = x \quad (2)$

At C $\quad CR = CQ = 12 - x \quad (3)$

Here, $AB = 5$ cm, $BC = 12$ cm and $\Delta B = 90^{\circ}$

Now $\quad AC = \sqrt{12^2 + 5^2} = \sqrt{144 + 25}$

$= \sqrt{169} = 13$ cm

Now $\quad AC = AR + RC$

$13 = 5 - x + 12 - x$

$2x = 17 - 13 = 4$

$x = \frac{4}{2} = 2$ cm

Hence, radius of the circle is 2 cm.

Q6.

Given, O is the centre of the given circle

$\therefore$ OQ and OP are the radius of circle.

$\because$ PT is a tangent

$$\therefore \quad OP \perp PT$$

So, $\angle OPT = 90°$

$$\therefore \quad \angle OPQ = 90° - \angle QPT$$

$$\Rightarrow \quad \angle OPQ = 90° - 60°$$

[Given, $\angle QPT = 60°$]

$$\Rightarrow \quad \angle OPQ = 30°$$

$$\therefore \quad \angle OQP = 30°$$

[$\because$ ΔOPQ is isosceles triangle]

Now, in ΔOPQ

$$\angle POQ + \angle OPQ + \angle OQP = 180°$$

$$\Rightarrow \quad \angle POQ + 30° + 30° = 180°$$

$$\Rightarrow \quad \angle POQ = 120°$$

reflex $\angle POQ = 360° - 120° = 240°$

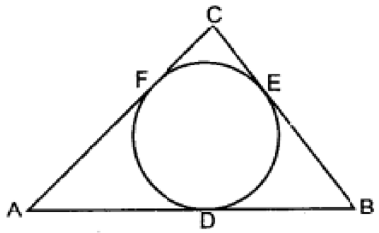
$$\therefore \quad \angle PRQ = \frac{1}{2}\text{reflex } \angle POQ$$

[$\because$ The angle substended by an arc of a circle at the centre is double the angle substended by it at any point on the remaining part of the circle]

$$\Rightarrow \quad \angle PRQ = \frac{1}{2} \times 240°$$

Hence, $\angle PRQ = 120°$

Q7.



Solution:

Given, AB = 12 cm, BC = 8 cm, AC = 10 cm

Let	$AD = x$ cm	
$\therefore$	$BD = AB - AD = (12 - x)$ cm	
$\because$	$AD = AF$	[tangents from point A]
$\therefore$	$AF = x$ cm	
Now,	$CF = AC - AF = (10 - x)$ cm	
Also,	$CE = CF \Rightarrow CE = (10 - x)$ cm	
And	$BD = BE$	[$\because$ tangents from B]
$\Rightarrow$	$BE = (12 - x)$ cm	[From (i)]
Now,	$BC = CE + BE$	
$\Rightarrow$	$8 = (10 - x) + (12 - x)$	
$\Rightarrow$	$8 = 22 - 2x \Rightarrow 2x = 14$	
$\Rightarrow$	$x = 7$ cm	
$\Rightarrow$	$AD = 7$ cm	
	$BE = 12 - x = 12 - 7 = 5$ cm	
and	$CF = 10 - x = 10 - 7 = 3$ cm	

Q8.

$$AB = AC$$

$\therefore$ $AR + BR = AQ + CO$

$AR + BR = AR + CQ$

[AQ = AR, euqal tangents]

$\Rightarrow$ $BR = CQ$

Now, $BR = BP$ [Length of equal tangents]

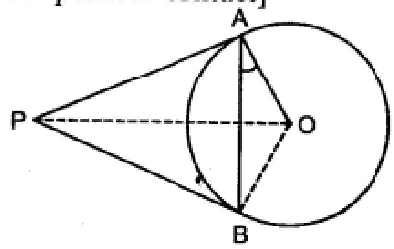
$CQ = CP$

$\Rightarrow$ $BP = CP$

Q9.

Construction: Join OP and OB.

Proof: Now, $OA \perp AP$ [Radius is perpendicular to tangent at the point of contact]



$\Rightarrow \quad \angle OAP = 90°$

Similarly, $\quad OB \perp BP$

$\Rightarrow \quad \angle OBP = 90°$

In quadrilateral OAPB

$$\angle OAP + \angle OBP = 90° + 90° = 180°$$

$\therefore$ Quadrilateral OAPB is a cyclic quadrilateral.

$\Rightarrow \quad \angle OAB = \angle OPB$ [Angles in same segment] ...(*i*)

Also, $\quad \angle OPB = \angle OPA$ [OP bisects $\angle APB$]

$\Rightarrow \quad \angle OPB + \angle OPA = 2\angle OPB$ [From (*i*)]

$\Rightarrow \quad \angle APB = 2\angle OPB$ ...(*ii*)

From (*i*) and (*ii*)

$$\angle APB = 2\angle OAB \quad [\because \angle OAB = \angle OPB]$$

Q10:

We have, $PR = PQ$

and $\angle PRQ = \angle PQR$

In ΔPQR,

$\angle PRQ + \angle PQR + \angle RPQ = 180°$

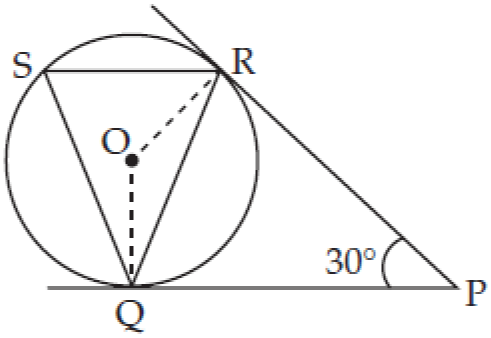
$\Rightarrow \quad 2\angle PRQ + 30° = 180°$

$$\Rightarrow \quad \angle PRQ = \frac{180° - 30°}{2} = 75°$$

$\because$ $SR \parallel QP$ and QR is a transversal

$\therefore \quad \angle SRQ = \angle PQR = 75°$

Join OR, OQ.



$\therefore \quad \angle ORQ = \angle RQO = 90° - 75° = 15°$

$\therefore \quad \angle QOR = (180° - 2 \times 15°)$

$= 180° - 30° = 150°$

$$\angle QSR = \frac{1}{2}\angle QOR$$

$= 75°$

(Angle subtended on arc is half the angle subtended on centre)

$\therefore$ In ΔSQR

$$\angle RQS = 180^\circ - (\angle SRQ + \angle RSQ)$$
$$= 180^\circ - (75^\circ + 75^\circ)$$
$$\therefore \quad \angle RQS = 30^\circ$$