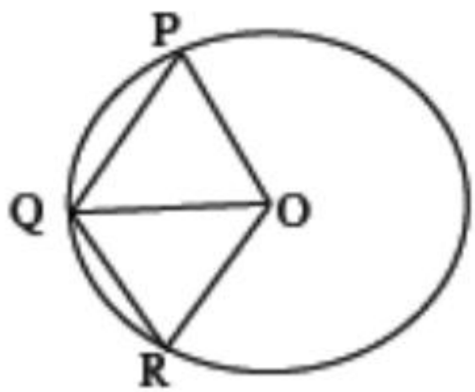
Chapter- Circles

Q1.

From a point P, two tangents PA are drawn to a circle with center O. If OP=diameter of the circle show that triangle APB is equilateral.

Q2.

In the given fig OPQR is a rhombus, three of its vertices lie on a circle with centre O If the area of the rhombus is $32\sqrt{3}$ cm^2. Find the radius of the circle.



Q3.

The radius of the in circle of a triangle is 4cm and the segments into which one side is divided by the point of contact are 6cm and 8cm. Determine the other two sides of the triangle.

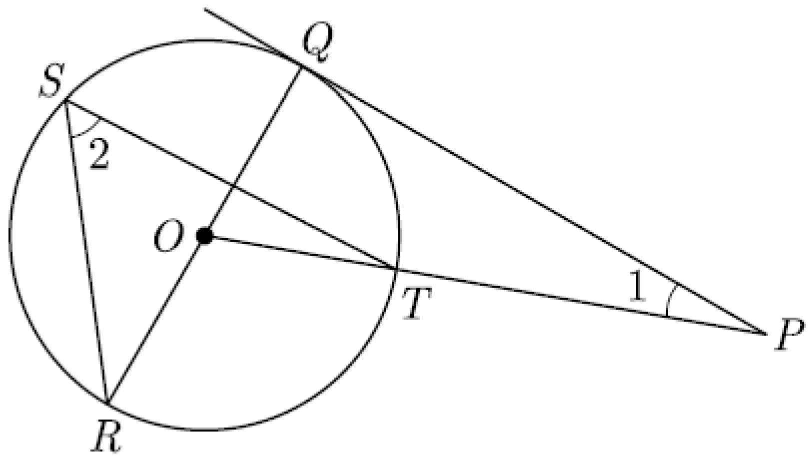
Q4.

13. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.

Q5. If d_1, d_2 ($d_2 > d_1$) be the diameters of two concentric circles and c be the length of a chord of a circle which is tangent to the other circle, prove that $(d_2)^2 = c^2 + (d_1)^2$.

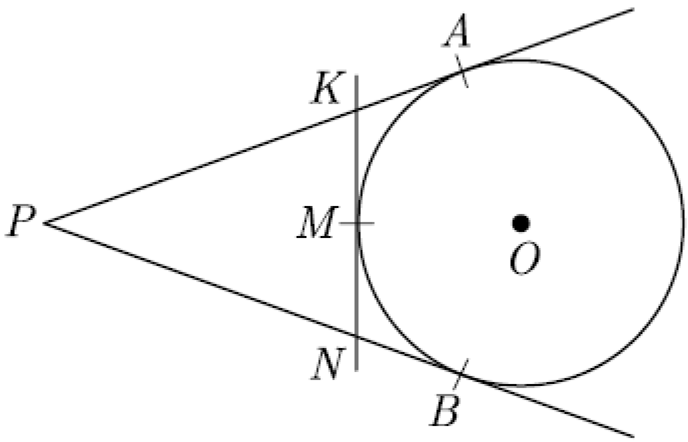
Q6.

.In figure PQ is a tangent from an external point P to a circle with centre O and OP cuts the circle at T and $\angle QOR$ is a diameter. If $\angle POR = 130^\circ$ and S is a point on the circle, find $\angle 1 + \angle 2$.



Q7.

.In given figure, PA and PB are tangents from a point P to the circle with centre O. At the point M, other tangent to the circle is drawn cutting PA and PB at K and N. Prove that the perimeter of $\Delta PNK = 2PB$.



Q8.

.Prove that the parallelogram circumscribing a circle is a rhombus.

Q9.

.From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of line segment PQ.

Q10.

If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that

$$AQ = \frac{1}{2}(BC + CA + AB)$$

Solutions

Q1.

Ans: PA=PB (length of tangents from an external point

From ΔOAP,

$\sin\angle APO = \frac{OA}{OP} = \frac{1}{2}$

Since $OP = 2OA$ (Since OP=Diameter)

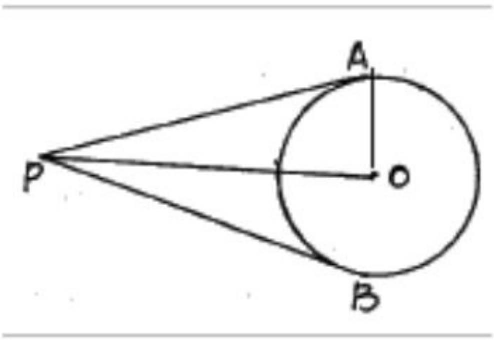
$\therefore \angle APO = 30^{o}$

since $\Delta\ APO \cong \Delta BPO$

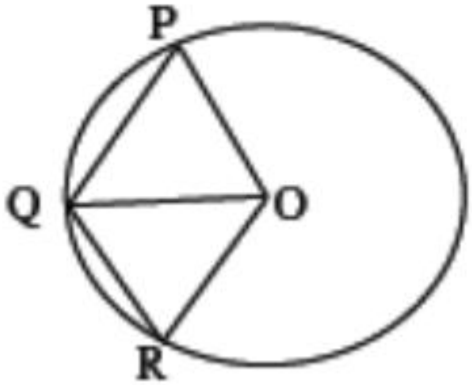
$\angle APO = \angle BPO = 30^{o}$

$\therefore \angle APB = 60^{o}$

ΔAPB is equilateral



Q2.



Ans: QP = OR

OP = OQ

$\therefore \Delta OPQ$ is a equilateral Δ.

area of rhombus = 2 (ar of ΔOPQ)

$$32\sqrt{3} = 2\left(\frac{\sqrt{3}r^2}{4}\right)$$

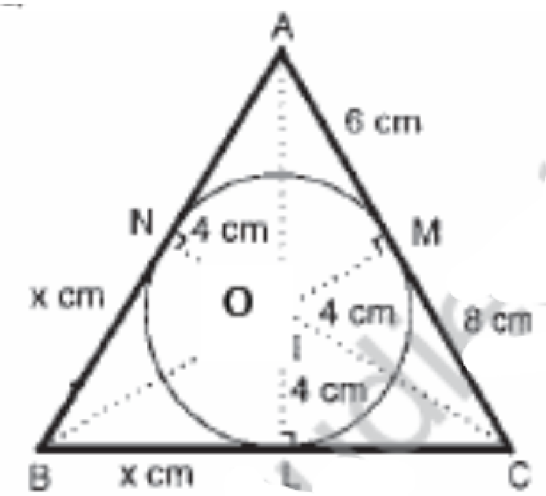
$$32\sqrt{3} = \frac{\sqrt{3}r^2}{2}$$

$r^2 = 32 \times 2 = 64$

$\Rightarrow r = 8cm$

$\therefore$ Radius = 8cm

Q3.



Ans: $a = BC = x + 8$

$b = AC = 6 + 8 = 14cm$

$c = AB = x + 6$

$\text{Semi} - \text{perimeter} = \dfrac{a+b+c}{2}$

$= \dfrac{BC + AC + AB}{2}$

$= \dfrac{x + 8 + 14 + x + 6}{2}$

$= \dfrac{2x + 28}{2}$

$= x + 14$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$ on substituting we get

$= \sqrt{(x+14)(6)(x)(8)}$

$= \sqrt{(x+14)(48x)}$ ....(1)

Area of ΔABC = area ΔAOB + area BOC + area Δ AOC

area $\Delta AOC = \left(\frac{1}{2}bh\right) = \frac{1}{2}$ x 4 x 14

= 28

On substituting we get

$\therefore$ area ΔABC = area ΔAOC + area ΔBOC + area ΔAOB

= 4x + 56 ....(2)

From (1) and (2)

$4x + 56 = \sqrt{(x+14)(48x)}$

Simplify we get $x = 7$

$\therefore AB = x + 6 = 7 + 6 =$ 13cm

$\therefore BC = x + 8 = 7 + 8 =$ 15cm

Q4.

Since $\Delta ADF \cong \Delta DFC$

$\angle ADF = \angle CDF$

$\therefore \angle ADC = 2\angle CDF$

Similarly we can prove $\angle CEB = 2\angle CEF$

Since $l \parallel m$

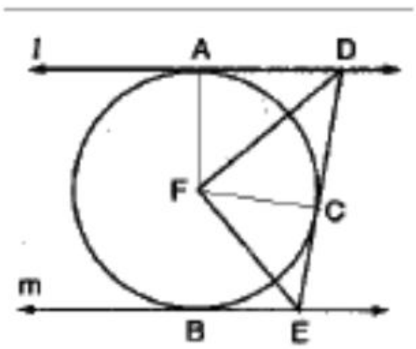
$\angle ADC + \angle CEB = 180^\circ$

$\Rightarrow 2\angle CDF + 2\angle CEF = 180^\circ$

$\Rightarrow \angle CDF + \angle CEF = 90^\circ$

In Δ DFE

$\angle DFE = 90^\circ$



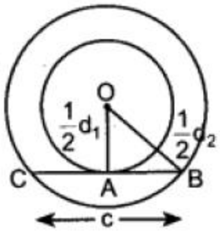
Q5.

$\because$ Diameter of bigger circle $= d_2$

So, Radius of bigger circle $= \frac{1}{2}d_2 = OB$

and Diameter of smaller circle $= d_1$

So, Radius of smaller circle $= \frac{1}{2}d_1 = OA$



$AB = \frac{c}{2}$ [$\because$ Diameter of CB = 'C']

In right ΔOAB, $\angle A = 90^\circ$

[$\because$ radius is perpendicular the tangent at point of contact]

By pythagoras theorem $OB^2 = AB^2 + OA^2$

$\Rightarrow \left(\frac{1}{2}d_2\right)^2 = \left(\frac{1}{2}c\right)^2 + \left(\frac{1}{2}d_1\right)^2 \Rightarrow \frac{1}{4}d_2^2 = \frac{1}{4}c^2 + \frac{1}{4}d_1^2$

$\Rightarrow d_2^2 = c^2 + d_1^2$ Hence proved.

Q6.

Here $\angle OQP = 90^\circ$ because radius is always perpendicular to tangent at point of contact.
Angle subtended at the centre is always 2 time of angle subtended at circumference by same arc. Thus

$$\angle 2 = \frac{1}{2}\angle TOR = \frac{1}{2}\angle POR$$

$$= \frac{1}{2} \times 130^\circ \quad 65^\circ$$

Now $\angle POQ = 180^\circ - 130^\circ = 50^\circ$

$$\angle 1 = 180^\circ - \angle OQP - \angle POQ$$

$$= 180^\circ - 90^\circ - 50^\circ = 40^\circ$$

Now $\angle 2 + \angle 1 = 65^\circ + 40^\circ = 105^\circ$

Q7.

Since length of tangents from an external point to a circle are equal,

$$
\begin{aligned}
PA &= PB \\
KM &= KA \\
MN &= BN
\end{aligned}
$$

Now
$$
\begin{aligned}
KN &= KM + MN \\
&= KA + BN
\end{aligned}
$$

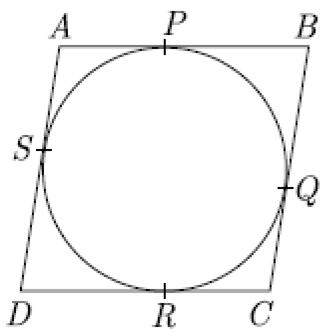
Now perimeter of $\Delta\, PNK$

$$
\begin{aligned}
p &= PN + KN + PK \\
&= PN + BN + KA + PK \\
&= PB + PA \\
&= 2PB \qquad (PA = PB)
\end{aligned}
$$

Q8.

Let $ABCD$ be the parallelogram.

$$AB = CD, AD = BC \qquad (1)$$



Since length of tangents from an external point to a circle are equal,

At A, $\quad AP = AS \qquad (2)$

At B $\quad BP = BQ \qquad (3)$

At C $\quad CR = CQ \qquad (4)$

At D $\quad DR = DS \qquad (5)$

Adding above 4 equation we have

$$AP + PB + CR + DR = AS + BQ + CQ + DS$$

or, $\quad AB + CD = AD + BC$

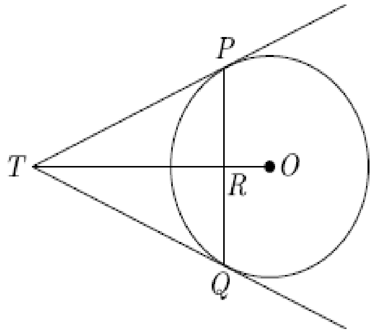
From (1) $\quad 2AB = 2AD$

or $\quad AB = AD$

Thus $ABCD$ is a rhombus.

Q9.

A circle with centre O. Tangents TP and TQ are drawn from a point T outside a circle as shown in figure below.



Since length of tangents from an external point to a circle are equal,

$$TP = TQ$$

Angle $\angle TPR$ and $\angle TQR$ are opposite angle of equal sides, thus

$$\angle TPR = \angle TQR$$

Now in ΔPTR and ΔQTR

$$TP = TQ$$

$$TR = TR \qquad \text{(Common)}$$

$$\angle TPR = \angle TQR$$

Thus $\quad \Delta PTR \cong \Delta QTR$

and $\quad PR = QR$

and $\quad \angle PRT = \angle QRT$

But $\angle PRT + QRT = 180^\circ$ as PQ is line segment,

$$\angle PRT = \angle QRT = 90^\circ$$

Therefore TR or OT is the right bisector of line segment PQ.

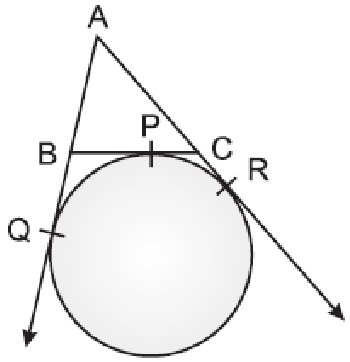
Q10.

We know that the lengths of tangents drawn from an external point to a circle are equal.

$$\therefore \quad AQ = AR \quad ...(i) \text{ [Tangents from A]}$$

$$BP = BQ \quad ...(ii) \text{ [Tangents from B]}$$

$$CP = CR \quad ...(iii) \text{ [Tangents from C]}$$



We have to show that

$$AQ = \frac{1}{2}(BC + CA + AB)$$

Taking L.H.S.

$$AQ = \frac{1}{2} \times 2AQ$$

$$\Rightarrow \quad AQ = \frac{1}{2} \times (AQ + AQ)$$

$$\Rightarrow \quad AQ = \frac{1}{2} \times (AQ + AR)$$

[From equation (i)]

$$\Rightarrow \quad AQ = \frac{1}{2} \times (AB + BQ + AC + CR)$$

$$\Rightarrow \quad AQ = \frac{1}{2} \times (AB + BP + AC + CP)$$

[Using equations (ii) and (iii)]

$$\Rightarrow \quad AQ = \frac{1}{2} \times \{AB + (BP + CP) + AC\}$$

$$\Rightarrow \quad AQ = \frac{1}{2} \times (AB + BC + AC)$$

Hence Proved.