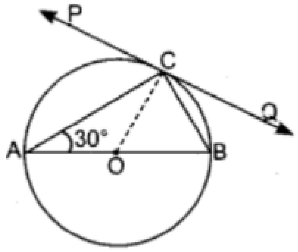
Chapter- Circles

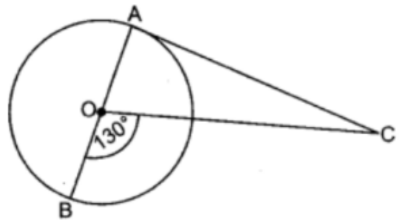
Q1.

In given figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle CAB = 30°$, find $\angle PCA$



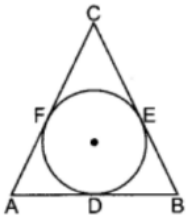
Q2.

In figure given, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130°$, then find $\angle ACO$.



Q3.

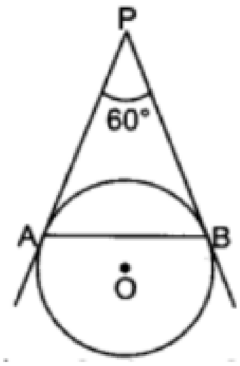
In given figure, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF



Q4.

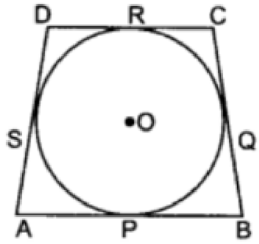
If given figure, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and

$\angle APB = 60°$. Find the length of chord AB.



Q5.

In figure, a quadrilateral ABCD is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively. Prove that AB + CD = BC + DA.

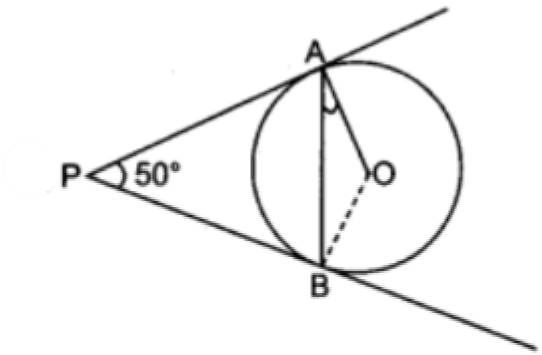


Q6.

Prove that the lengths of tangents drawn from an external point to a circle are equal

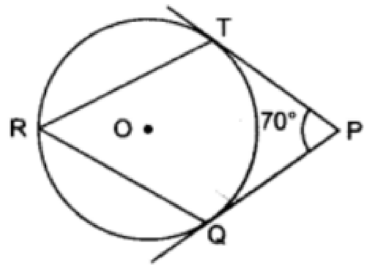
Q7.

In figure, PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$. Write the measure of $\angle OAB$



Q8.

In figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If ∠TPQ = 70°, find ∠TRQ

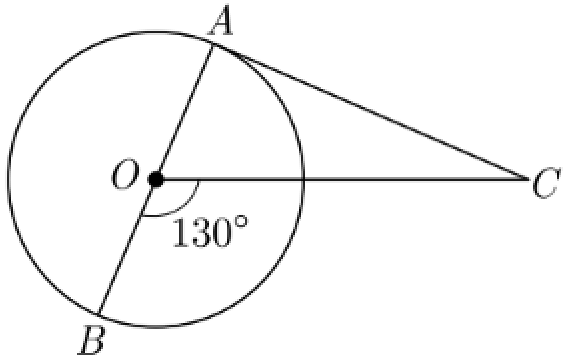


Q9.

. What is the length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm ?

Q10.

In the given figure, AOB is a diameter of the circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130°$, the find $\angle ACO$.



Solution 1.

Construction: Join AO.

Given: PQ is tangent. AB is diameter $\angle CAB = 30°$.

To Find: $\angle PCA$

Solution: In ΔAOC, $AO = CO$ ($\because$ Equal radii)

$\angle CAO = \angle OCA$ ($\because$ Angles opposite to equal sides are equal)

or $\angle CAB = \angle OCA$

But, $\angle CAB = 30°$ So, $\angle OCA = 30°$ (*i*)

Since, $OC \perp PQ$ ($\because$ Tangent is perpendicular to radius at point of contact)

$\Rightarrow$ $\angle PCO = 90° \Rightarrow \angle OCA + \angle PCA = 90° \Rightarrow 30° + \angle PCA = 90°$

$\therefore$ $\angle PCA = 60°$

Solution 2.

$\angle AOC + \angle BOC = 180°$

[$\because$ Linear Pair Axiom]

$\angle AOC + 130° = 180°$

$\angle AOC = 180° - 130°$

$\angle AOC = 50°$

Now, $\angle OAC = 90°$ [angle between radius OA and tangent AC is 90°]

Now, in ΔAOC,

$\angle OAC + \angle AOC + \angle ACO = 180°$ [$\because$ sum of angles in triangle is 180°]

$90° + 50° + \angle ACO = 180°$

$\angle ACO = 180° - 140°$

$\angle ACO = 40°$

Solution 3.

Given, AB = 12 cm, CA = 10 cm, BC = 8 cm

Let $AD = AF = x$ [$\because$ Tangent drawn from external point to circle are equal]

$\therefore$ $DB = BE = 12 - x$ and $CF = CE = 10 - x$

$BC = BE + EC \Rightarrow 8 = 12 - x + 10 - x$

$\Rightarrow$ $x = 7$

$\therefore$ AD = 7 cm, BE = 5 cm and CF = 3 cm

Solution 4.

In ΔAPB we have $AP = BP$

$\Rightarrow \angle PAB = \angle PBA$

[$\because$ Tangents from an external point are equally inclined to segment joining centre to point]

Let $\angle PAB = x,$

then in ΔAPB, $x + x + 60^\circ = 180^\circ$

$$2x = 180^\circ - 60^\circ = 120^\circ$$

$$x = 60^\circ$$

As all three angles of ΔAPB are 60°. So ΔAPB is an equilateral triangle.

Hence $AP = BP = AB = 5$ cm

Solution 5.

We know that tangents drawn to a circle from an outer points are equal.

So, $AP = AS, BP = BQ,$

$$CR = CQ \text{ and } DR = DS.$$

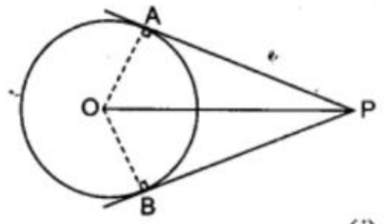
Now, consider

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$\Rightarrow$ $AB + CD = AD + BC$

Hence proved.

Solution 6.



Given: A circle $C(O, r)$, P is a point outside the circle and PA and PB are tangents to a circle.

To Prove: PA = PB

Construction: Draw OA, OB and OP.

Proof: Consider triangles OAP and OBP.

$$\angle OAP = \angle OBP = 90^\circ \quad ...(i)$$

[Radius is perpendicular to the tangent at the point of contact]

$$OA = OB \text{ (radii)} \quad ...(ii)$$

OP is common ...(*iii*)

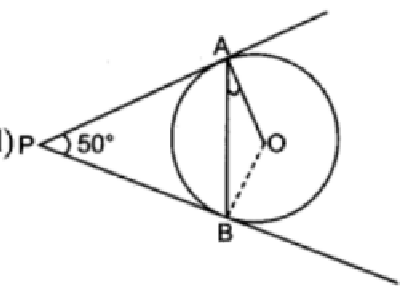
$\therefore$ $\Delta OAP \cong \Delta OBP$ (RHS) [from (*i*), (*ii*) and (*iii*)]

Hence, $AP = BP$ (CPCT)

Solution 7.

Join OB.

$\because$ PA and PB are tangents to the circle drawn from an external point P. We know that, tangent is perpendicular r to radius.



$$\angle OAP = \angle OBP = 90^\circ$$

Then, $\angle OAP + \angle APB + \angle OBP + \angle AOB = 360^\circ$

(ASP of quadrilateral)

$\therefore$ $\angle APB + \angle AOB = 180^\circ$

$\Rightarrow$ $50^\circ + \angle AOB = 180^\circ$

$\Rightarrow$ $\angle AOB = 130^\circ$

In ΔOAB, $OA = OB$ ($\because$ equal radii)

$\Rightarrow$ $\angle A = \angle B = x$ (say)($\because$ angles opposite to equal sides are equal)

$\angle A + \angle B + \angle AOB = 180^\circ$ ($\because$ ASP of triangles)

$\Rightarrow$ $x + x + 130^\circ = 180^\circ$

$\Rightarrow$ $2x = 50^\circ$

$\Rightarrow$ $x = 25^\circ$

$\therefore$ $\angle OAB = 25^\circ$

Solution 8.

We know that tangent is perpendicular to radius. Hence,

$$\angle OTP = \angle OQP = 90^\circ$$

In quadrilateral PQOT,

$$\angle QOT + \angle OTP + \angle TPQ + \angle OQP = 360^\circ \quad [\because \text{ ASP of quadrilateral}]$$

$$\angle TOQ + \angle TPQ = 180^\circ$$

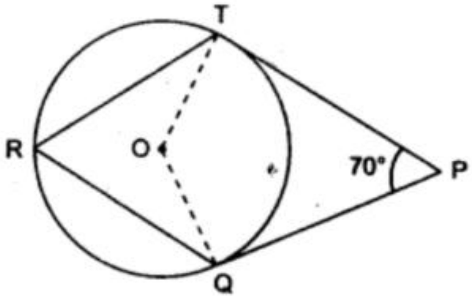
$\Rightarrow$ $\angle TOQ = 110^\circ$

Also $\angle TOQ = 2\angle TRQ$

[angle subtended by an arc at centre of the circle is twice the angle subtended by it in alternate segment]

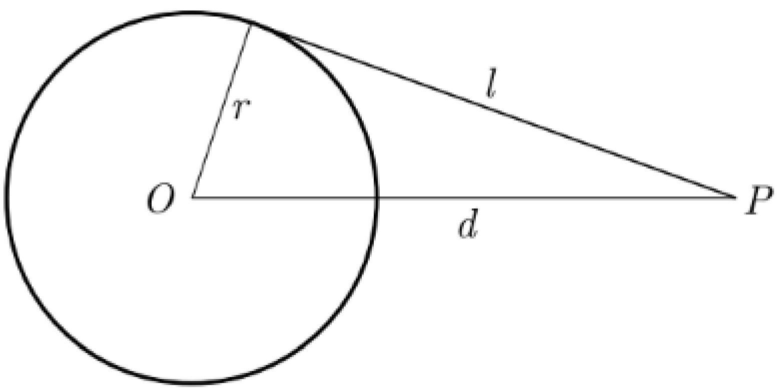
$\Rightarrow$ $110^\circ = 2\angle TRQ$

$\Rightarrow$ $\angle TRQ = 55^\circ$



Solution 9.

As per the given question we draw the figure as below.



Length of the tangent,

$$\begin{aligned} l &= \sqrt{d^2 - r^2} \\ &= \sqrt{8^2 - 6^2} \\ &= \sqrt{64 - 36} \\ &= \sqrt{28} = 2\sqrt{3} \text{ cm.} \end{aligned}$$

Solution 10.

Here OA is radius and AC is tangent at A, since radius is always perpendicular to tangent, we have

$$\angle OAC = 90^{\circ}$$

From exterior angle property,

$$\angle BOC = OAC + \angle ACO$$

$$130^{\circ} = 90^{\circ} + \angle ACO$$

$$\angle ACO = 130^{\circ} - 90^{\circ} = 40^{\circ}$$