Chapter-Arithmetic Progressions
Question bank

Q1.

If S_n, the sum of first n terms of an A.P. is given by $S_n = 3n^2 - 4n$, find the nth term. [CBSE Delhi, Set 1, 2019]

Q2

The sum of the first 7 terms of an A.P. is 63 and that of its next 7 terms is 161. Find the A.P. [CBSE Delhi, Set 3, 2020]

Q3.

Find the sum of all 11 terms of an A.P. whose middle term is 30. [CBSE OD, Set–II, 2020]

Q4.

Which term of the A.P. 20, $19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$, ... is the first negative term.

Q5.

.An arithmetic progression $5, 12, 19, \ldots\ldots$ has 50 terms. Find its last term. Hence find the sum of its last 15 terms.

Q6.

.If $1 + 4 + 7 + 10 \ldots\ldots + n = 287$, Find the value of n.

Q7.

.Find the value of a, b and c such that the numbers $a, 7, b, 23$ and c are in AP

Q8.

.The first term of an AP is 3, the last term is 83 and the sum of all its terms is 903. Find the number of terms and the common difference of the AP.

Q9.

.Find the sum of the two digits numbers divisible by 6.

Q10.

.If the ratio of the 11^{th} term of an AP to its 18^{th} term is 2 : 3, find the ratio of the sum of the first five term of the sum of its first 10 terms.

Solutions

Q1.

Given, $S_n = 3n^2 - 4n$

We know that

$$a_n = S_n - S_{n-1}$$
$$= 3n^2 - 4n - [3(n-1)^2 - 4(n-1)]$$
$$= 3n^2 - 4n - [3(n^2 - 2n + 1) - 4n + 4]$$
$$= 3n^2 - 4n - (3n^2 - 6n + 3 - 4n + 4)$$
$$= 3n^2 - 4n - 3n^2 + 10n - 7$$
$$= 6n - 7$$

So, nth term will be $6n - 7$

Q2.

For the given A.P.,
$S_7 = 63$ and $S_{14} - S_7 = 161$
Now we know,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad S_7 = \frac{7}{2}[2a + (7-1)d]$$

$$\Rightarrow \quad 63 = \frac{7}{2}[2a + 6d]$$

$$\Rightarrow \quad a + 3d = 9 \quad \text{...(i)}$$

Also, $S_{14} - S_7 = 161$

$$\Rightarrow \quad S_{14} = S_7 + 161$$

$$\Rightarrow \frac{14}{2}[2a + (14-1)d] = 63 + 161$$

$$\Rightarrow \quad 2a + 13d = \frac{224}{7}$$

$$\Rightarrow \quad 2a + 13d = 32$$

From equation (i) we get $a = 9 - 3d$

Putting this value of a in equation (ii), we ge

$$2(9 - 3d) + 13d = 32$$

$$\Rightarrow \quad 18 - 6d + 13d = 32$$

$$\Rightarrow \quad 7d = 32 - 18$$

$$\Rightarrow \quad 7d = 14$$

$$\Rightarrow \quad d = 2$$

$$\Rightarrow \quad a = 9 - 3(2) \quad \text{...(from eq (i}$$

$$\Rightarrow \quad a = 3$$

$$a + d = 3 + 2 = 5$$

$$a + 2d = 3 + 2(2) = 7$$

∴ A.P. is 3, 5, 7 **Ans.**

Q3.

Total number of terms = 11 *i.e.*, odd

$$\therefore \text{ Middle term } = \left(\frac{11+1}{2}\right)^{\text{th}} \text{ term } = 6^{\text{th}} \text{ term}$$

$$\Rightarrow \quad a_6 = 30$$

Let a be the first term and d be the common difference.

Then, $a + 5d = 30$ ...(i)

Now, $S_{11} = \frac{11}{2}[2a + (11 - 1)d]$

$$S_{11} = \frac{11}{2}[2a + 10d]$$

$$S_{11} = 11\,[a + 5d]$$

$$S_{11} = 11 \times 30 \quad \text{[Using (i)]}$$

$$= 330$$ **Ans.**

$\therefore$ First term $(a) = 20$

and common difference (d)

$$= 19\frac{1}{4} - 20$$

$$= \frac{77}{4} - 20 = \frac{77-80}{4} = -\frac{3}{4}$$

Let n^{th} term be the first negative term

So, $a_n < 0$

$\Rightarrow$ $a + (n-1)d < 0$

$\Rightarrow$ $20 + (n-1)\left(-\frac{3}{4}\right) < 0$

$\Rightarrow$ $20 - \frac{3n}{4} + \frac{3}{4} < 0$

$\Rightarrow$ $\frac{83}{4} - \frac{3n}{4} < 0$

$\Rightarrow$ $\frac{3n}{4} > \frac{83}{4}$

$\Rightarrow$ $3n > 83$

$\Rightarrow$ $n > \frac{83}{3}$

$\Rightarrow$ $n > 27\frac{2}{3}$

So, $n = 28$

Hence, 28^{th} term is the first negative term.

Ans.

Q4.

Q5.

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.
We have $a = 5$, $d = 12 - 5 = 7$ and $n = 50$

$$\begin{aligned} a_{50} &= 5 + (50-1)7 \\ &= 5 + 49 \times 7 = 348 \end{aligned}$$

Also the first term of the AP of last 15 terms be a_{36}

$$\begin{aligned} a_{36} &= 5 + 35 \times 7 \\ &= 5 + 245 = 250 \end{aligned}$$

Now, sum of last 15 terms,

$$\begin{aligned} S_{36-50} &= \frac{15}{2}[a_{36} + a_{50}] \\ &= \frac{15}{2}[250 + 348] \\ &= \frac{15}{2} \times 598 = 4485 \end{aligned}$$

Hence, sum of last 15 terms is 4485.

Q6.

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.
We have $a=1, d=3$ and $S_n=287$.

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\frac{n}{2}[2\times 1+(n-1)3] = 287$$

$$\frac{n}{2}[2+(3n-3)] = 287$$

$$3n^2-n = 574$$

$$3n^2-n-574 = 0$$

$$3n^2-42n+41n-574 = 0$$

$$3n(n-14)+41(n-14) = 0$$

$$(n-14)(3n+41) = 0$$

Since negative value is not possible, $n = 14$

$$a_{14} = a+(n-1)d$$

$$= 1+13\times 3 = 40$$

Q7.

Let the common difference be d.

Since $a, 7, b, 23$ and c are in AP, we have

$$a+d=7$$

..(1)

$$a+3d=23 \quad \ldots(2)$$

Form equation (1) and (2), we get

$$a=-1, d=8$$

$$b=a+2d=-1+2\times 8=-1+16=15$$

$$c=a+4d=-1+4\times 8=-1+32=31$$

Thus $\quad a=-1, b=15, c=31$

Q8.

First term, $a = 3$

Last term, $a_n = 83$

Sum of n terms, $S_n = 903$

Since, $S_n = \frac{n}{2}(a + a_n)$

$$903 = \frac{n}{2}(3 + 83)$$

$$1806 = 86n$$

$$n = \frac{1806}{86} \Rightarrow n = 21$$

Now $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$903 = \frac{21}{2}[2 \times 3 + (21 - 1)d]$$

$$1806 = 21(6 + 20d)$$

$$6 + 20d = 86$$

$$20d = 80 \Rightarrow d = 4$$

Hence, the common difference is 4.

Q9.

Series of two digits numbers divisible by 6 is 12, 18, 24, ...96. It forms an AP. Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 12, d = 18 - 12 = 6, a_n = 96$

$$a_n = a + (n-1)d$$

$$96 = 12 + (n-1) \times 6$$

$$84 = 6(n-1)$$

$$n = 14 + 1 = 15$$

$$S_n = \frac{n}{2}[a + a_n]$$

$$= \frac{15}{2}[12 + 96]$$

$$= \frac{15 \times 108}{2}$$

$$= 15 \times 54 = 810$$

Q10.

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Now $$\frac{a_{11}}{a_{18}} = \frac{a+10d}{a+17d} = \frac{2}{3}$$

$$2(a+17d) = 3(a+10d)$$

$$a = 4d \qquad ...(1)$$

Now, $$\frac{S_5}{S_{10}} = \frac{\frac{5}{2}(2a+4d)}{\frac{10}{2}[2a+9d]} = \frac{(a+2d)}{[2a+9d]}$$

Substituting the value $a = 4d$ we have

or, $$\frac{S_5}{S_{10}} = \frac{4d+2d}{8d+9d} = \frac{6}{17}$$

Hence $S_5 : S_{10} = 6:17$