Chapter-Arithmetic Progressions
Question bank

Q1.

Let $\{a_n\}$ be a non-constant arithmetic progression. $a_1 = 1$ and the following holds true: for any $n \geq 1$, the value of $\frac{a_{2n} + a_{2n-1} + ... + a_{n+1}}{a_n + a_{n-1} + ... + a_1}$ remains constant (does not depend on n). Find a_{15}

Q2.

If the mth term of an AP is a and its nth term is b, show that the sum of its (m + n) terms is

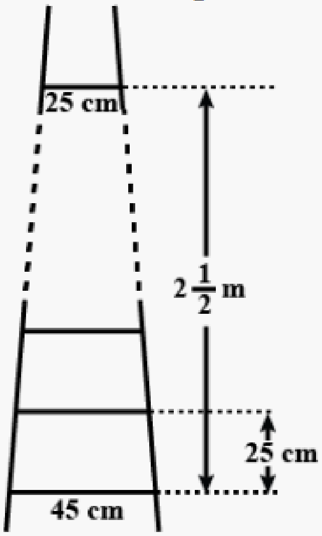
$$\frac{(m+n)}{2}\{a + b + \frac{(a-b)}{(m-n)}\}$$

Q3.

.An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the past three terms is 429. Find the AP.

Q4.

A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$m apart, what is the length of the wood required for the rungs



Q5.

Balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row of two balls and so on. If 669 more balls are added then all the balls can be arranged in the shape of a square and each of the sides then contains 8 balls less than each side of the triangle did. Determine the initial numbers of balls.

Q6.

If m times the m^{th} term of an A.P. is equal to n times n^{th} term, show that the $(m + n)^{th}$ term of the A.P. is zero.

Q7.

In an AP, if the pth term is 1/q and qth term is 1/ p. Find the sum of first pq term.

Q8.

Show that the sum of (m + n)th and (m - n)th terms of an A.P. is equal to twice the mth terms.

Q9.

Show that the sum of an AP whose first term is a, the second term b and the last term is c is equal to $\frac{(a + c)(b + c - 2a)}{2(b - a)}$

Q10.

.Find $\left(4-\frac{1}{n}\right)+\left(7-\frac{2}{n}\right)+\left(10-\frac{3}{n}\right)+\ldots\ldots$ upto n terms.

Solutions

Q1.

Let us denote $C=\frac{a_{2n}+a_{2n-1}+\ldots+a_{n+1}}{a_n+a_{n-1}+\ldots+a_1}$. By adding 1 to both sides, we get $C+1=\frac{a_{2n}+a_{2n-1}+\ldots+a_{n+1}}{a_n+a_{n-1}+\ldots+a_1}+\frac{a_n+a_{n-1}+\ldots+a_1}{a_n+a_{n-1}+\ldots+a_1}=\frac{a_{2n}+a_{2n-1}+\ldots+a_1}{a_n+a_{n-1}+\ldots+a_1}=\frac{S_{2n}}{S_n}$. Since C is constant, then $C+1$ is also constant. Therefore, $\frac{S_{2n}}{S_n}$ must not depend on n. But

$\frac{S_{2n}}{S_n}=\frac{\frac{2a_1+(2n-1).d}{2}.2n}{\frac{2a_1+(n-1).d}{2}n}=2\frac{2+(2n-1)d}{2+(n-1)d}$.

Let us denote $\frac{C+1}{2}=R$, which is also a constant. We have

$\frac{2+(2n-1)d}{2+(n-1)d}=R$

$2R+(n-1).d.R=2+(2n-1).d$

$2R+n.d.R-d.R=2+2n.d-d$

$n.d(R-2)=d.R-2R-d+2=R(d-2)-(d-2)=(R-1)(d-2)$. This must hold true for any n. Let us assume that both sides are non-zero. The left side changes with n, while the right side does not, which means they will be unequal at some point. Therefore both sides must be zero. n and d are non-zero (since $\{a_n\}$ is a non-constant progression), so for the left side to be zero, R must be 2. Which leads to $0=1\times(d-2)$, which means that d must also be 2. The arithmetic progression is now defined, with $a_1=1$, $d=2$ and $a_{15}=a_1+14\times d=1+14\times 2=29$

Q2.

Given,

a + (m – 1)d = a

a + (n – 1)d = b

a – b = [a + (m – 1)d] – [a + (n – 1)d]

a – b = (m – n)d

$\therefore d=\frac{a-b}{m-n}$

$S_n=\frac{n}{2}[2a+(n-1)d]$

$S_{m+n}=\frac{m+n}{2}[2a+(m+n-1)d]$

$=\frac{m+n}{2}[2a+(m+n-2)d+d]$

$=\frac{m+n}{2}[a+(m-1)d+a+(n-1)d+d]$

$=\frac{m+n}{2}[a+b+d]$

$\therefore S_n=\frac{m+n}{2}[a+b+\frac{a-b}{m-n}]$

Hence proved.

Q3.

Let the middle most terms of the AP be $(x-d)$, x and $(x+d)$.

We have $x-d+x+x+d=225$

$$3x=225 \Rightarrow x=75$$

and the middle term $=\frac{37+1}{2}=19^{th}$ term

Thus AP is

$(x-18d),\ldots.(x-2d),(x-d),x,(x+d),(x+2d),\ldots\ldots.$

$(x-18d)$

Sum of last three terms,

$$(x+18d)+(x+17d)+(x+16d)=429$$

$$3x+51d=429$$

, $$225+51d=429 \Rightarrow d=4$$

First term $a_1=x-18d=75-18\times 4=3$

$$a_2=3+4=7$$

Hence AP = 3, 7, 11, ..., 147.

Q4.

It is given that the rungs are 25 cm apart and the top and bottom rungs are $2\frac{1}{2}$ m apart.

$\therefore$ the total number of rungs.

$$\frac{2\frac{1}{2}\times 100}{25}+1$$

$$=\frac{250}{25}+1=11$$

Now, as the lengths of the rungs decrease uniformly, they will be in an A.P.

The length of the wood required for the rungs equals the sum of all the terms of this A.P.

First term, $a = 45$

Last term, $l = 25$

$n = 11$

$$S_n = \frac{n}{2}(a+l)$$

$$S_{10} = \frac{11}{2}(45+25) = \frac{11}{2}\times 70 = 385 \text{ cm}$$

Therefore, the length of wood is 385cm

Q5.

Solution

Verified by Toppr

Let the number of rows in which the balls are arranged to form an equilateral triangle be n.

According to the given condition the total number of balls

$$S = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

(since number of balls in the kth row of equilateral triangle is k)

$$\therefore S + 669 = (n-8)^2$$

$$\Rightarrow \frac{n(n+1)}{2} + 669 = n^2 - 16n + 64$$

$$\Rightarrow n^2 + n + 1338 = 2n^2 - 32n + 128$$

$$\Rightarrow n^2 - 33n - 1210 = 0$$

$$\Rightarrow n = \frac{33 \pm \sqrt{1089 + 4840}}{2}$$

$$\Rightarrow n = \frac{33 \pm 77}{2}$$

$$\Rightarrow n = 55$$

Therefore initial number of balls

$$= \frac{n(n+1)}{2} = \frac{55 \times 56}{2} = 1540$$

Q6.

Solution

Given,

nth term of AP = $t_n = a + (n - 1)d$

mth term of AP = $t_m = a + (m - 1)d$

$\Rightarrow mt_m = nt_n$

$m[a + (m - 1)d] = n[a + (n - 1)d]$

$m[a + (m - 1)d] - n[a + (n - 1)d] = 0$

$a(m - n) + d[(m + n)(m - n) - (m - n)] = 0$

$(m - n)[a + d((m + n) - 1)] = 0$

$a + [(m + n) - 1]d = 0$

But $t_{m+n} = a + [(m + n) - 1]d$

$\therefore t_{m+n} = 0$

Q7.

Given $a_p = \frac{1}{q}$

$a + (p - 1)d = \frac{1}{q}$

$aq + (pq - q)d = 1 \ldots\ldots(1)$

Similarly, we get

$ap + (pq - p)d = 1 \ldots\ldots(2)$

From (1) and (2), we get

$aq + (pq - q)d = ap + (pq - p)d$

$aq - ap = d[pq - p - pq + q]$

$a(q - p) = d(q - p)$

$\therefore a = d$

Equation (1) becomes,

$dq + pqd - dq = 1$

$d = \frac{1}{pq}$

Hence, $a = \frac{1}{pq}$

Consider,

$S_{pq} = \frac{pq}{2}[2a + (pq - 1)d]$

$= \frac{pq}{2}[2\left(\frac{1}{pq}\right) + (pq - 1)\left(\frac{1}{pq}\right)]$

$= \frac{1}{2}[2 + pq - 1]$

$= \frac{1}{2}[pq + 1]$

Q8.

Let a_n be the first term and 'd' be the common difference of the given A.P.

$\therefore a_{m+n} = a + (m + n - 1)d$ ..... (1)

$a_{m-n} = a + (m - n - 1)d$ ..... (2)

Adding (1) and (2)

$a_{m+n} + a_{m-n} = a + (m + n - 1)d + a + (m - n - 1)d$

$= a + md + nd - d + a + md - nd - d$

$= 2a + 2md - 2d$

$= 2(a + md - d)$

$= 2[a + (m - 1)d] = 2[a_m]$

Thus, sum of the (m + n)th and (m - n)th terms of an A.P. is equal to twice the mth term.

Q9.

First term = a

second term = b Last term = c

Difference d = b – a

In an AP, n^{th} term $T_n = a + (n - 1)d$

$= a + (n - 1)(b - a)$

$c = a + (n - 1)(b - a)$

$\therefore c - a = (n - 1)(b - a)$

$\therefore n - 1 = \dfrac{(c - a)}{(b - a)}$

$\therefore n = \dfrac{c - a}{b - a} + 1$

Sum of n numbers = n/2 (first term+last term)

$= n/2(a + c)$

Q10.

Let sum of n term be S_n, then we have

$$s_n = \left(4 - \frac{1}{n}\right) + \left(7 - \frac{2}{n}\right) + \left(40 - \frac{3}{n}\right) + \text{..... upto } n \text{ terms.}$$

$$= (4 + 7 + 10 + \text{.....} + n \text{ terms}) - \left(\frac{1}{n} + \frac{2}{n} + \frac{3}{n} \text{.....} + 1\right)$$

$$= (4 + 7 + 10 + \text{....} + n \text{ terms}) - \frac{1}{n}(1 + 2 + 3 + \ldots n)$$

$$= \frac{n}{2}[2 \times 4 + (n-1)(3)] - \frac{1}{n} \times \frac{n}{2}[2 \times 1 + (n-1)(1)]$$

$$= \frac{n}{2}[8 + 3n - 3] - \frac{1}{2}[2 + n - 1]$$

$$= \frac{n}{2}(3n + 5) - \frac{1}{2}(n + 1)$$