Chapter-Areas Related to Circles

1. Circumference and Area of a Circle

The circumference is the distance around the circle, while the area is the space enclosed by the circle.

- Circumference of a circle: $2\pi r$
- Area of a circle: πr^2

Example 1: Find the circumference and area of a circle with radius 7 cm.
Solution:
Circumference = $2\pi r = 2 \times \frac{22}{7} \times 7 = 44\,\text{cm}.$
Area = $\pi r^2 = \frac{22}{7} \times 7^2 = 154\,\text{cm}^2.$

Example 2: The circumference of a circle is 44 cm. Find its radius and area.
Solution:
Circumference = $2\pi r = 44.$
$r = \frac{44}{2\pi} = \frac{44}{2\times\frac{22}{7}} = 7\,\text{cm}.$
Area = $\pi r^2 = \frac{22}{7} \times 7^2 = 154\,\text{cm}^2.$

2. Area of a Sector

A sector is the region enclosed by two radii and the arc between them. Its area depends on the central angle (θ).

Area of a sector: $\frac{\theta}{360^\circ} \times \pi r^2$, where θ is the central angle.

Example 1: Find the area of a sector with radius 6 cm and central angle 60°.
Solution:
Area of sector = $\frac{\theta}{360^\circ} \times \pi r^2 = \frac{60}{360} \times \frac{22}{7} \times 6^2 = 18.85\,\text{cm}^2$.

Example 2: A circular park has a radius of 14 m. A sprinkler waters an area forming a sector with a central angle of 90°. Find the watered area.
Solution:
Area of sector = $\frac{\theta}{360^\circ} \times \pi r^2 = \frac{90}{360} \times \frac{22}{7} \times 14^2 = 154\,\text{m}^2$.

3. Length of an Arc

The length of an arc is part of the circumference corresponding to a central angle (θ°)

Length of an arc: $\frac{\theta}{360^\circ} \times 2\pi r$

Example 1: Find the length of an arc in a circle with radius 21 cm and central angle 60°.
Solution:
Length of arc = $\frac{\theta}{360^\circ} \times 2\pi r = \frac{60}{360} \times 2 \times \frac{22}{7} \times 21 = 22\,\text{cm}$.

Example 2: A wheel with radius 35 cm rolls forward such that its arc subtends an angle of 45° at the center. Find the distance covered by the wheel.
Solution:
Length of arc = $\frac{\theta}{360^\circ} \times 2\pi r = \frac{45}{360} \times 2 \times \frac{22}{7} \times 35 = 27.5\,\text{cm}$.

4. Area of a Segment

A segment is the region enclosed by an arc and its chord. Its area is calculated as:

$$\text{Area of segment} = (\text{Area of sector}) - (\text{Area of triangle})$$

Example 1: A chord divides a circle with radius 14 cm into two segments, subtending an angle 60° at the center. Find the area of the minor segment.
Solution:
Area of sector = $(\frac{\theta}{360})\pi r^2 = (\frac{60}{360}) \times \frac{22}{7} \times 14^2 = 102.67\,\text{cm}^2$.
Area of triangle (equilateral): $(\sqrt{3}/4)a^2,\ a = 14 = 84.87\,\text{cm}^2$.
Area of segment = $102.67 - 84.87 = 17.8\,\text{cm}^2$.

Example 2: A circular pizza with radius 28 cm is divided into four equal slices. Each slice is further divided into two segments by cutting along chords parallel to their bases. Find the area of one segment if each slice subtends an angle of 90° at the center.
Solution:

1. Area of one slice (sector): $\frac{90}{360} \times \pi \times 28^2 = 615.44\,\text{cm}^2$
2. Area of triangle in one slice:
   - Base = $28\sqrt{2} = 39.60\,\text{cm}$
   - Height = $28\,\text{cm}$
   - Area = $\frac{1}{2} \times 39.60 \times 28 = 554.40\,\text{cm}^2$
3. Area of segment = $\frac{615.44-554.40}{2} = 30.52\,\text{cm}^2$