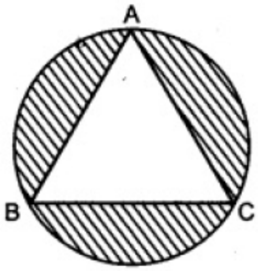
Chapter-Areas Related to Circles

Q1.

In given figure, an equilateral triangle has been inscribed in a circle of radius 6 cm. Find the area of the shaded region.



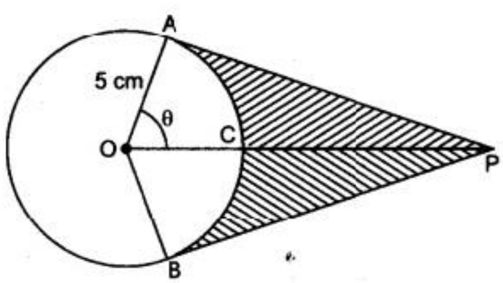
Q2.

The long and short hand of a clock are 6cm and 4 cm long respectively,Find the sum of the distance travelled by their tips in 24hrs.

Q3.

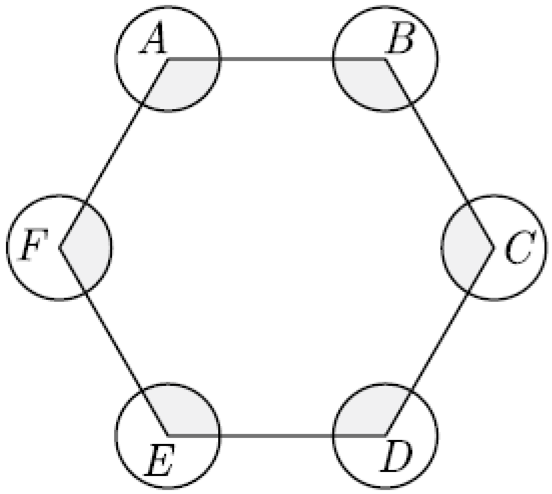
An elastic belt is placed around the rim of a pulley of radius 5 cm. From one point C on the belt, the elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley.

(use π = 3.14



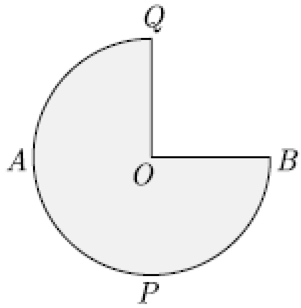
Q4.

.In fig., $ABCDEF$ is any regular hexagon with different vertices A, B, C, D, E and F as the centres of circle with same radius r are drawn. Find the area of the shaded portion.



Q5.

In fig. APB and AQP are semi-circle, and $AO = OB$. If the perimeter of the figure is 47 cm, find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Q6.

. Three horses are tied each with 7 m long rope at three corners of a triangular field having sides 20 m, 34 m and 42 m. Find the area of the plot which can be grazed by the horses.

Q7.

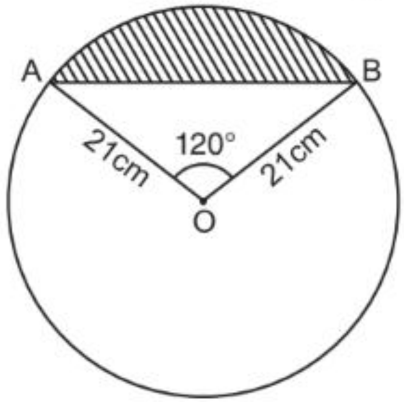
The diameters of the front and rear wheels of a tractor are 80 cm and 200 cm respectively. Find the number of revolutions of rear wheel to cover the distance which the front wheel covers in 800 revolutions.

Q8.

. The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

Q9.

Find the area of the segment shown in Fig. if radius of the circle is 21 cm and $\angle AOB = 120°$ $\left(\text{Use } \pi = \frac{22}{7}\right)$



Q10.

A piece of wire 22 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Solutions

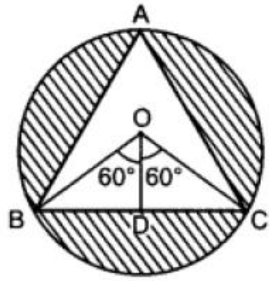
Q1.

Solution:

ΔABC is equilateral,

$\therefore \quad \angle BOC = 120°$ (Angle subtended by chord at centre is double the angle subtended by the same chord at the circle)

Construction: Draw $OD \perp BC$.



So, $\angle BOD = 60°$

In ΔOBD, $\cos 60° = \frac{OD}{OB}$ and $\sin 60° = \frac{BD}{OB}$

$\Rightarrow \quad \frac{1}{2} = \frac{OD}{6}$ and $\frac{\sqrt{3}}{2} = \frac{BD}{6}$

$\Rightarrow \quad \frac{6}{2} = OD$ and $\frac{6\sqrt{3}}{2} = BD$

$\Rightarrow \quad OD = 3$ and $BD = 3\sqrt{3}$

$BC = 2BD = 2 \times 3\sqrt{3} = 6\sqrt{3}$

Area of the shaded region = area of circle – area of ΔABC

$$= \pi(6)^2 - \frac{\sqrt{3}}{4}(6\sqrt{3})^2 = 3.14 \times 6 \times 6 - \frac{\sqrt{3}}{4} \times 6\sqrt{3} \times 6\sqrt{3}$$

$$= 113.04 - 27\sqrt{3} \text{ cm}^2$$

$$= 113.04 - 27(1.73) = 113.04 - 46.71 = 66.33 \text{ cm}^2$$

Q2.

Solution:

Distance covered by the tip of long hand in one hour

$=$ circumference of the circle with radius 6 cm

$= 2\pi \times 6 = 12\pi$ cm

$\therefore$ Distance travelled by long hand in 24 hours $= 24 \times 12\pi = 288\pi$ cm

Distance travelled by tip of short hand in 12 hrs

$=$ circumference of the circle with radius 4 cm

$= 2\pi \times 4 = 8\pi$ cm

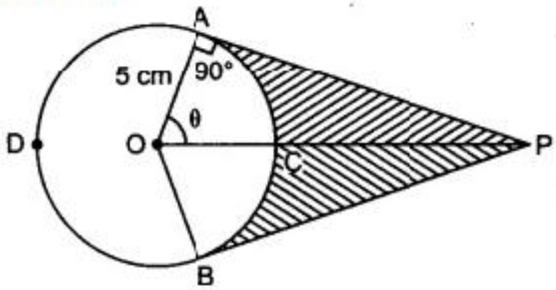
$\therefore$ Distance travelled by short hand in 24 hours $= 2 \times 8\pi = 16\pi$ cm

Total distance travelled $= 288\pi + 16\pi = 304\pi$ cm

$= 304 \times 3.14$ cm $= 954.56$ cm

Q3.

Solution:



Given: AO = 5 cm and OP = 10 cm

In right ΔAOP,

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$= \frac{AO}{OP} = \frac{5}{10} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 60^\circ$$

$$\angle AOB = \theta' = 2 \times 60 = 120^\circ$$

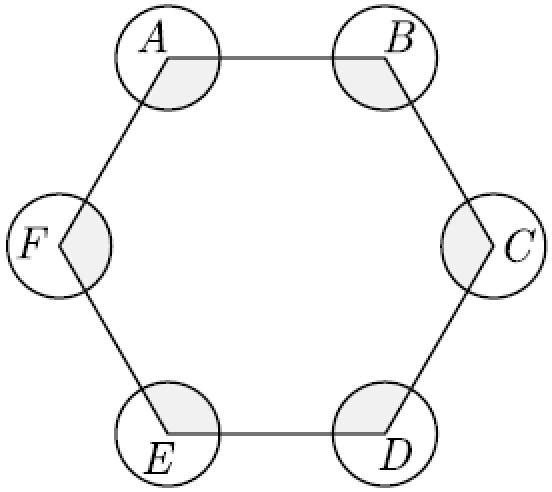
$$\text{Length of ADB} = \frac{360^\circ - \theta'}{360^\circ} \times 2\pi r$$

$$= \frac{240}{360} \times 2 \times 3.14 \times 5$$

$$= \frac{2}{3} \times 10 \times 3.14 = 20.93 \text{ cm}$$

Hence, length of belt in contact = 20.93 cm

Q4.



Ans :

Let n be number of sides.

Now $n \times$ each angle $= (n-2) \times 180^\circ$

$6 \times$ each angle $= 4 \times 180^\circ$

each angle $= 120^\circ$

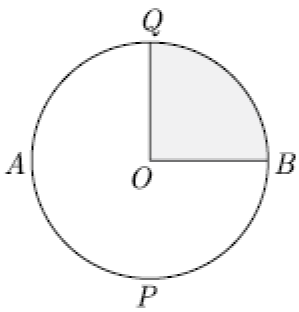
Area of a sector $= \pi r^2 \times \dfrac{120^\circ}{360^\circ}$

Area of 6 shaded regions $= 6\pi r^2 \times \dfrac{120^\circ}{360^\circ}$

$= 2\pi r^2$

Q5.

We have redrawn the given figure as shown below;



Let r be the radius of given circle. It is given that perimeter of given figure is 47 cm.

$$2\pi r - \frac{1}{4}(2\pi r) + 2r = 47$$

$$\frac{3\pi r}{2} + 2r = 47$$

$$r\left(\frac{3}{2} \times \frac{22}{7} + 2\right) = 47$$

$$r\left(\frac{33}{7} + 2\right) = 47$$

$$r = \frac{47 \times 7}{47} = 7 \text{ cm}$$

Now, area of shaded region

$$A = \text{area of circle} - \frac{1}{4} \text{ area of circle}$$

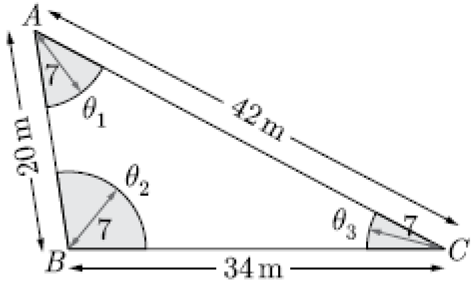
$$= \frac{3}{4} \text{ area of circle}$$

$$= \frac{3}{4}\pi r^2 = \frac{3}{4} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{3}{2} \times 77 = 115.5 \text{ cm}^2$$

Q6.

As per information given in question we have drawn the figure below.



Let $\angle A = \theta_1$, $\angle B = \theta_2$ and $\angle C = \theta_3$.

Now, area which can be grazed by the horses is the sum of the areas of three sectors with central angles θ_1, θ_2 and θ_3 each with radius $r = 7$ m.

$$\frac{\pi r^2 \theta_1}{360^\circ} + \frac{\pi r^2 \theta_2}{360^\circ} + \frac{\pi r^2 \theta_3}{360^\circ} = \frac{\pi r^2}{360^\circ}(\theta_1 + \theta_2 + \theta_3) \qquad ...(1)$$

From angle sum property of a triangle we have

$$\theta_1 + \theta_2 + \theta_3 = 180^\circ$$

Substituting above in equation (1) we have

$$\frac{\pi r^2 \theta_1}{360^\circ} + \frac{\pi r^2 \theta_2}{360^\circ} + \frac{\pi r^2 \theta_3}{360^\circ} = \frac{\pi r^2}{360^\circ} \times 180^\circ = \frac{\pi r^2}{2}$$

$$= \frac{22}{7} \times \frac{1}{2} \times (7)^2$$

$$= \frac{22}{7} \times \frac{1}{2} \times 7 \times 7$$

$$= 77 \text{ m}^2$$

Hence, the area grazed by the horses is 77 m^2

Q7.

Circumference of front wheel

$$\pi d = \frac{22}{7} \times 80 = \frac{1760}{7} \text{ cm}$$

Distance covered by front wheel in 800 revolutions

$$= \frac{1760}{7} \times 800$$

Circumference of rear wheel

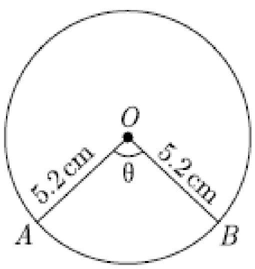
$$= \frac{22}{7} \times 200 \quad \frac{4400}{7} \text{ cm}$$

Revolutions made by rear wheel

$$= \frac{\frac{1760}{7} \times 800}{\frac{4400}{7}} = \frac{1760 \times 800}{4400} = 320 \text{ revolutions}$$

Q8.

From the given information we have drawn the figure as below.



Perimeter of the sector

$$p = 2r + \frac{2\pi r\theta}{360^\circ}$$

$$16.4 = 2 \times 5.2 + \frac{2\pi \times 5.2 \times \theta}{360^\circ}$$

$$16.4 = 10.4 + \frac{2\pi \times 5.2 \times \theta}{360^\circ}$$

$$6 = \frac{2\pi \times 5.2 \times}{360^\circ}$$

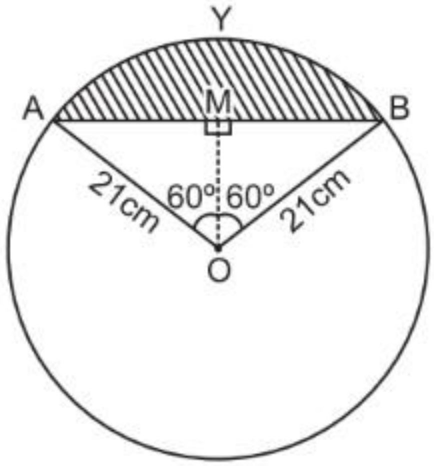
$$\frac{3}{5.2} = \frac{\theta \times \pi}{360^\circ}$$

Now, area of sector $= \frac{\theta}{360^\circ} \times \pi r^2 = \left(\frac{\theta \times \pi}{360^\circ}\right) r^2$

$$= \frac{3}{5.2} \times (5.2)^2 = 15.6 \text{ sq. units.}$$

Q9.

Given, Radius of the circle = 21 cm and $\angle AOB = 120°$



Area of the segment AYB

= Area of sector AOB – Area of ΔAOB

$$\text{Area of sector AOB} = \frac{120°}{360°} \times \frac{22}{7} \times 21 \times 21$$

$$= 462 \text{ cm}^2$$

To find the area of ΔOAB, draw $OM \perp AB$

$\Delta AMO \cong \Delta BMO$ (by R.H.S.)

$$\therefore \angle AOM = \angle BOM = \frac{1}{2} \times 120° = 60°$$

From ΔOMA, $\dfrac{OM}{OA} = \cos 60°$

$$\frac{OM}{21} = \frac{1}{2}$$

$$OM = \frac{21}{2}\text{ cm}$$

Also, $$\frac{AM}{OA} = \sin 60°$$

$$AM = 21 \times \frac{\sqrt{3}}{2}$$

or $$AB = 2 \times AM = 21\sqrt{3}\text{ cm}$$

So, $$\text{area of } \Delta OAB = \frac{1}{2} \times AB \times OM$$

$$= \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$$

$$= \frac{441}{4}\sqrt{3}\text{ cm}^2$$

$$\therefore \text{ Area of segment} = \left(462 - \frac{441}{4}\sqrt{3}\right)\text{cm}^2$$

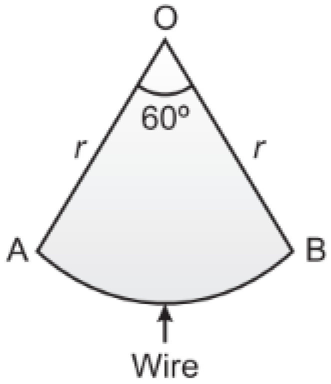
$$= \frac{21}{4}(88 - 21\sqrt{3})\text{ cm}^2$$

$$= 271.04\text{ cm}^2$$

Ans.

Q10.

Let the radius of the arc of the circle be r cm



We have

Length of wire (l) $= 22$ cm

and, $\theta = 60°$

We know,

$$\text{Length of arc/ wire} = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times r$$

$$= \frac{22}{21} r$$

$$\therefore \quad \frac{22}{21} r = 22$$

$$\Rightarrow \quad r = 22 \times \frac{21}{22}$$

$$\Rightarrow \quad r = 21 \text{ cm}$$ **Ans.**