CLASS 10TH MID TERM

SCORE BOOSTER

MATHS

POLYNOMIALS

Definition

Polynomials :- Algebraic Expressions consisting constants & variables Ex:

$2, 2x + 2, 2x^2 + 2x + 2$ *etc*.

Types of Polynomials:

- On basis of degree
- On the basis of number of terms

More examples of Polynomials

(variable's power is whole number)

$\sqrt{x^4} = x^2$

$2 \rightarrow 2x^0$

$2(1)$

$p(x) = \sqrt{3}x^2 + 5x^1 + 6x^3 - 9$ ✓

degree = 3

$p(x) = \sqrt{3x^1} + 2$

$= \sqrt{3} \cdot \sqrt{x} + 2$

$= \sqrt{3}(x)^{1/2} + 2$ ✗

$\sqrt{y}, \sqrt[3]{y}, \sqrt[9]{y}$

$(y)^{1/2}, (y)^{1/3}, (y)^{1/9}$

Classification on the basis of number of terms

- Monomial: A Polynomial containing one term

  ex: 2, 2x, $5y^2$

- Binomial: A polynomial containing two terms

  ex: x + 2, $2x^3 + x^5$

- Trinomial: A polynomial containing three terms

  ex: $x - x^2 + 8, -x^5 + x + x^4$

Classification on the basis of degree

- Constant polynomial: Polynomial with degree Zero

  $2, \quad 1795(1) = 1795(x^0) \to 0$

- Linear Polynomial: Polynomial with degree one

  $p+1, \quad t+5, \quad 5x+2$

- Quadratic Polynomial: Polynomial with degree two

  $s^2-4, \quad x^2+5x-2, \quad y^2-y$

- Cubic polynomial: Polynomial with degree three

  $x^3-x, \quad x^3-2x^2+x-2$

-
- **Zero polynomial:**

  $0 \to$ not defined

  $0(1) = 0x^0 \to$ degree '0'

  $= 0x^{1000}$ degree 1000

What is the value of a polynomial?

$P(x) = 5x + 2$

$x = 0 \rightarrow p(0) = 5(0) + 2$

$= 2$

$p(2) = 5(2) + 2$

$= 10 + 2 = 12$

$P(x) = x^2 - x + 2$

$p(0) = 0^2 - 0 + 2 = 2$

$p(2) = 2^2 - 2 + 2$

$= 4 - 2 + 2$

$= 4$

What are the zeroes of a polynomial?

$(x+2)(x-1)=0$

$x+2=0 \Rightarrow x=-2$

$x-1=0 \Rightarrow x=1$

$P(x) = 5x + 2 = 0$

$x = \frac{-2}{5}$

$5x+2=0$

$5x=-2$

$x=-2/5$

$-2, 1$

$P(x) = x^2 + 1x - 2 = 0$

$\Rightarrow x^2+2x-1x-2=0$

$x(x+2)-1(x+2)=0$

$(x+2)(x-1)=0$

$-2x^2$

$2x \quad -1x$

Important points to remember about zeroes of a polynomial?

1. Number of zeroes of a polynomial cannot exceed the degree of polynomial.

$x^3 - x + x^2 + 2$ $\quad x^{10} + \ldots$

2. Constant polynomial does not have a zero.

$p(x) = 2x^0$

$= 2(2)^0$

$= 2(1) = 2$

Geometrical meaning of zeroes of a polynomial

$P(x) = x + 2 \rightarrow$ line

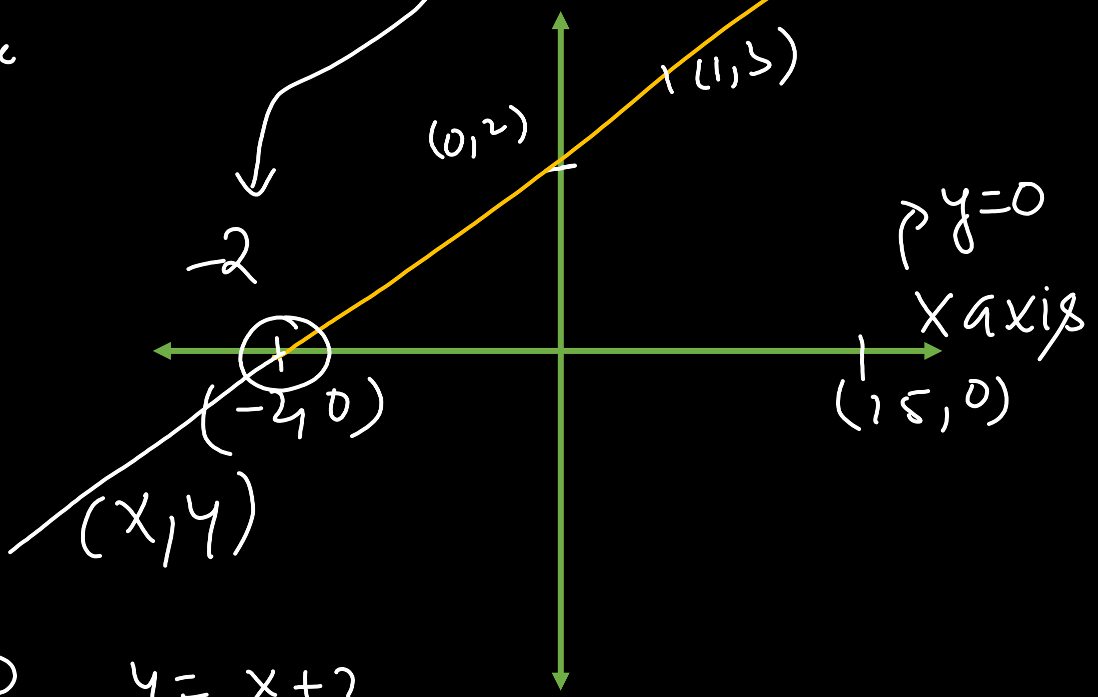
$y = (x + 2)$

$1 + 2 \rightarrow 3$

$y = 0$

$x = -2, y = 0$

Zero



$y = x + 2$

y	2	3	0
x	0	1	-2

Geometrical meaning of zeroes of a polynomial

$P(x) = x^2 + x - 2$

$= (x+2)(x-1) = 0$

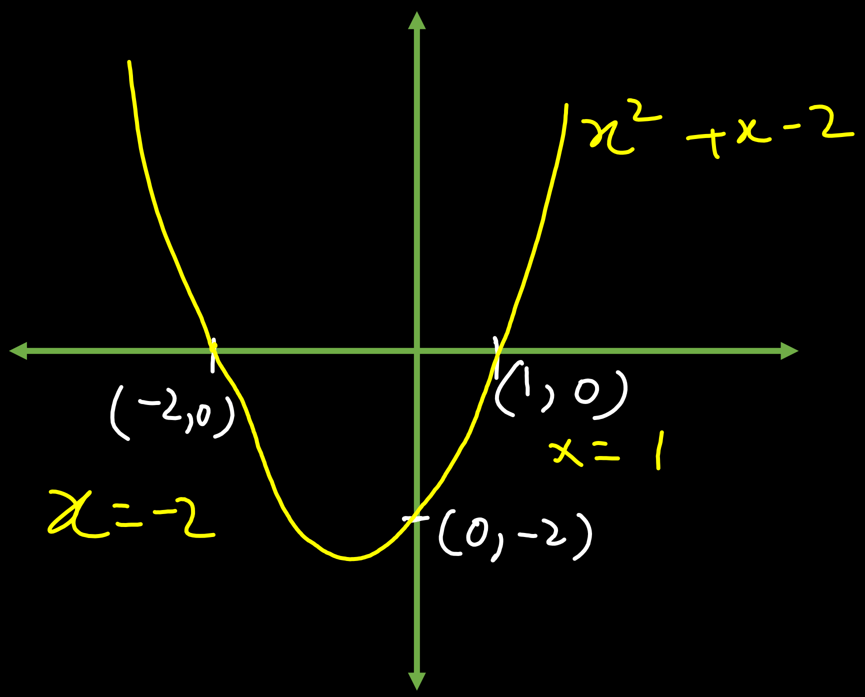
$x = -2$

$x = 1$

$p(0) = 0 + 0 - 2$

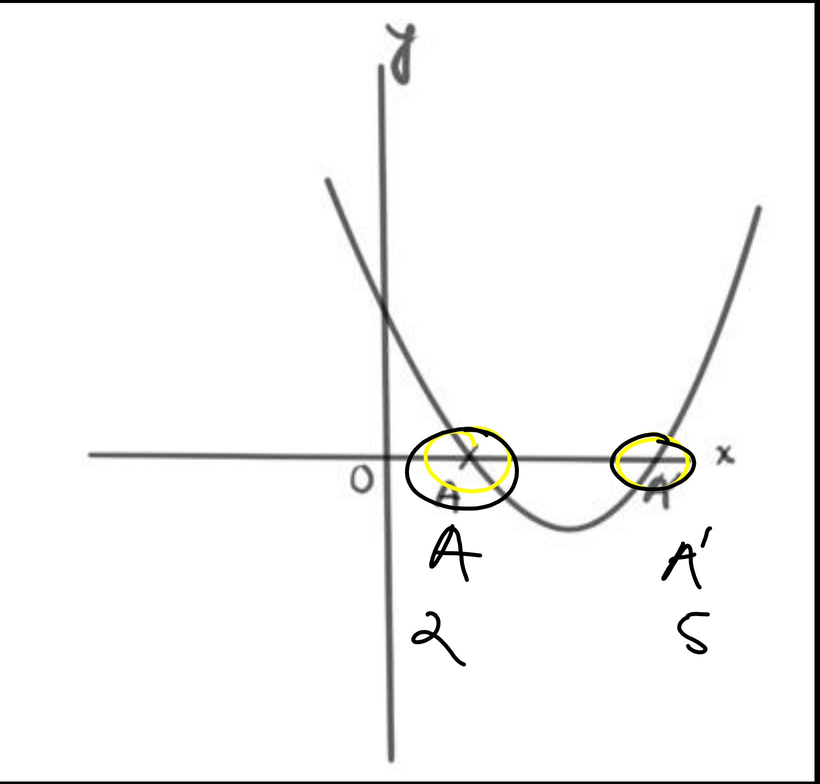
$= -2$

$y = x^2 + x - 2$



Number of zeroes of a polynomial (from graph)

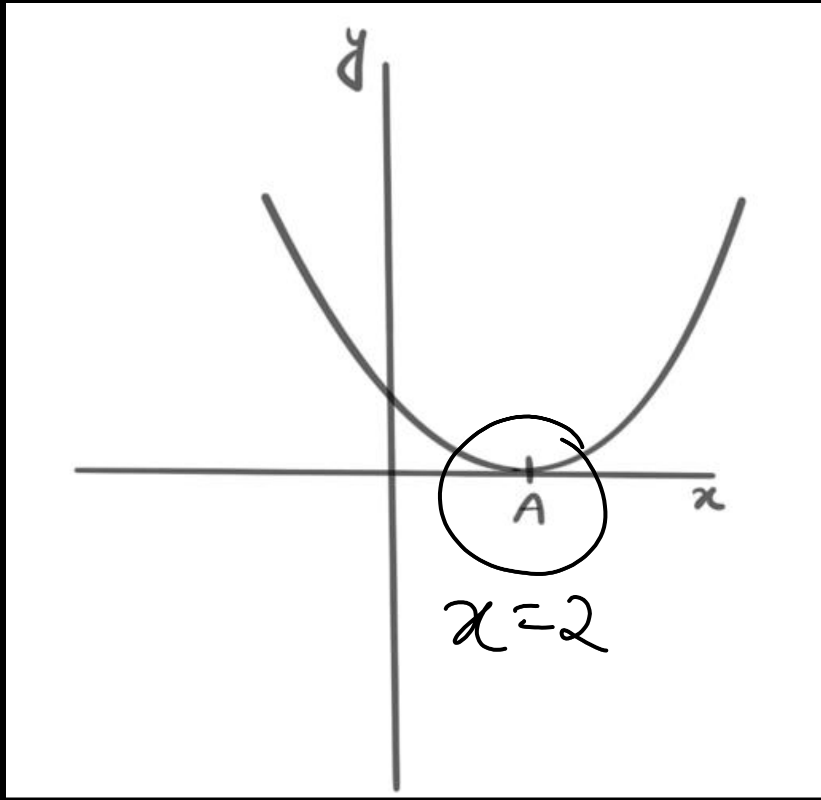
Case (i)



x = 2
x = 5

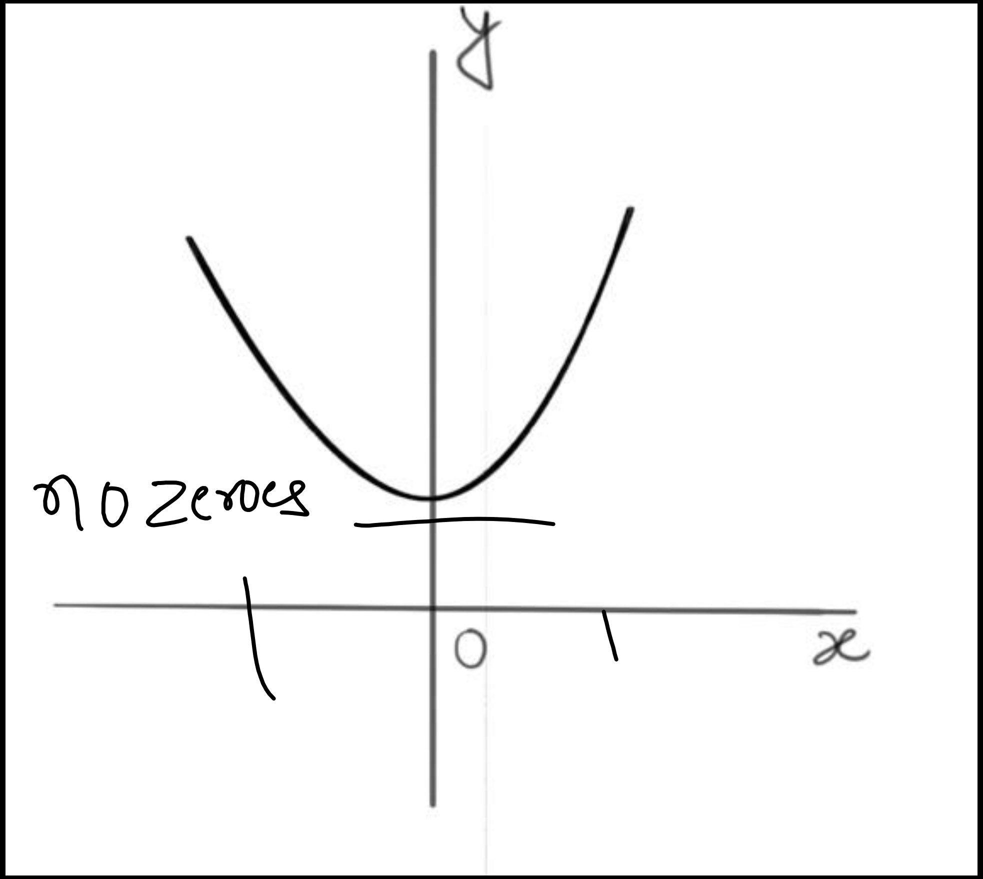
Number of zeroes of a polynomial (from graph)

Case (ii)

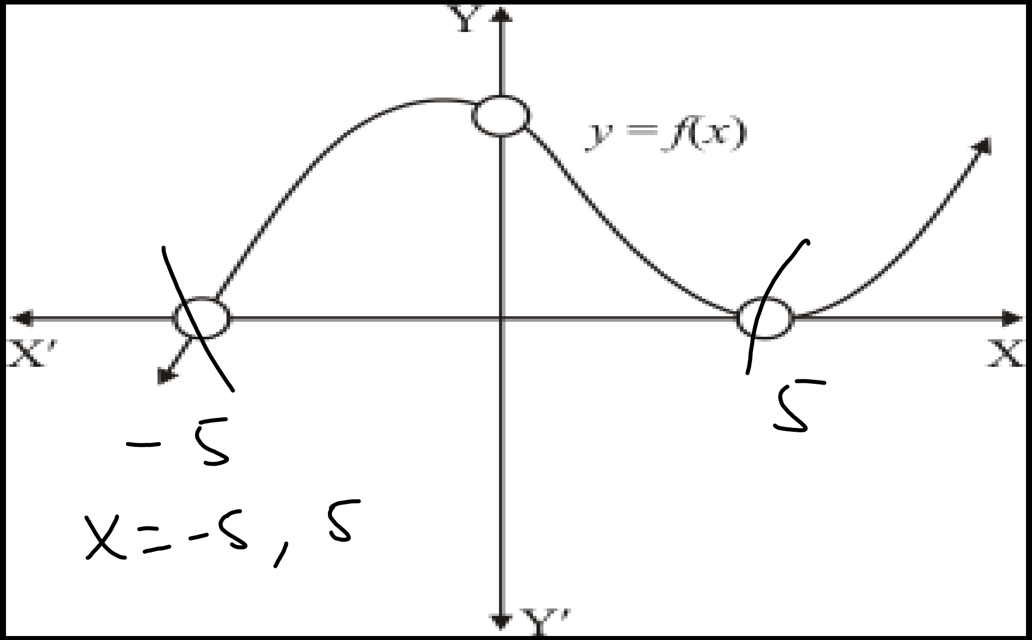


Number of zeroes of a polynomial (from graph)

Case (iii)

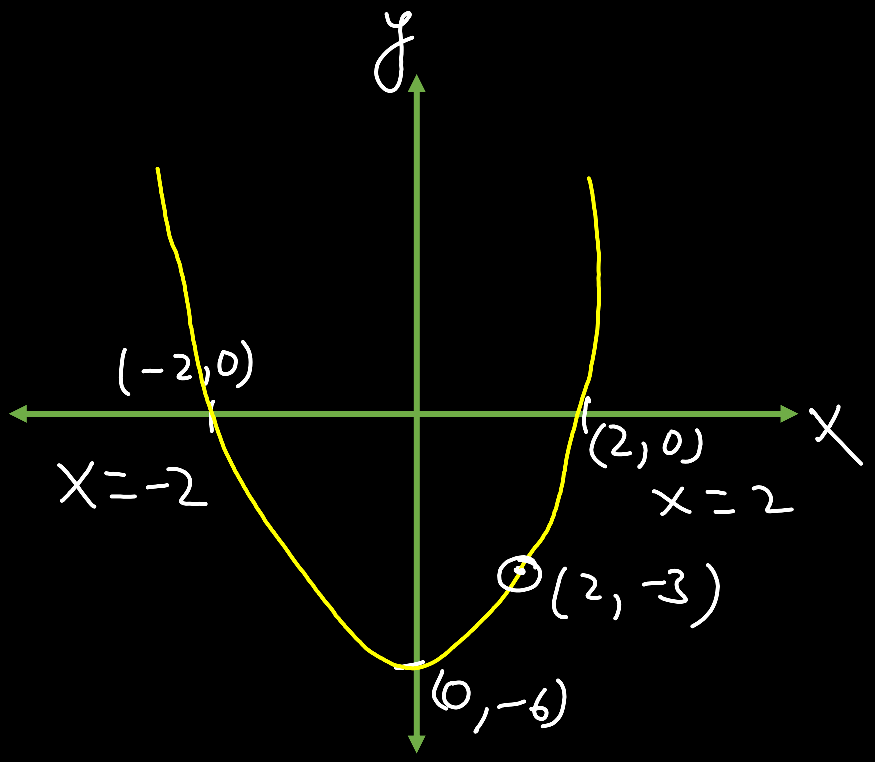


Q. A graph of a polynomial f(x) is shown in the figure find the number of zeroes.



Q. The graph of the quadratic polynomial p(x) passes through the point (-2, 0), (0, -6), (2, -3) and (2, 0). Find the zeroes of the polynomial.

$x = -2, 2$



Q. If one of the zero of quadratic polynomial $(k-1)x^2 + kx + 1$ is -3, the find k.

$x = -3$

$f(-3) = 0$

$f(x) = (k-1)x^2 + kx + 1$

$f(-3) = (k-1)(-3)^2 + k(-3) + 1$

$0 = (k-1)9 - 3k + 1$

$0 = 9k - 9 - 3k + 1$

$0 = 6k - 8$

$\frac{8}{6} = k \Rightarrow k = \frac{4}{3}$

Q. If the zeroes of quadratic polynomial $x^2 + (a+1)x + b$ are 2 & -3, then find a & b. → 0, −6

$\Rightarrow f(x) = x^2 + (a+1)x + b$

$f(2) = 2^2 + (a+1)2 + b$

$0 = 4 + 2a + 2 + b$

$-6 = 2a + b$ —— (1)

$f(-3) = (-3)^2 + (a+1)(-3) + b$

$0 = 9 - 3a - 3 + b$

$-6 = -3a + b$ —— (2)

eqn (1) − (2)

$-6 = 2a + b$

$-6 = -3a + b$

$0 = 5a$

$a = 0$

$-6 = 2a + b$

$-6 = 0 + b$

$b = -6$

Quadratic polynomial

Standard form ← $ax^2 + bx + c, a \neq 0$

$0 + bx + c = bx + c$

a :- Coeff. of x^2

b :- Coeff. of x

c :- Constant term

Examples:

$5x^2 + 13x - 17$ (a, b, c)

$3x^2 + x \rightarrow 3x^2 + 1x + 0$

$a = 3,\ b = 1,\ c = 0$

$-12x^2 + 2 \rightarrow -12x^2 + 0x + 2$ (a) (b) (c)

Relation between zeroes & coefficients of a quadratic polynomial

2 Zeros

For a quadratic polynomial

$ax^2 + bx + c$

Let the zeroes of the polynomial be α & β

Sum of zeroes $= \alpha + \beta = \left(-\frac{b}{a}\right)$

Product of zeroes $= \alpha . \beta = \frac{c}{a}$

Q. Find the zeroes of the polynomial $x^2 + \frac{1}{6}x - 2$, and verify the relation between the coefficients and the zeroes of the polynomial.

$a = 1$
$b = \frac{1}{6}$
$c = -2$

$\Rightarrow x^2 + \frac{1}{6}x - 2 = \frac{6}{6}\left(x^2 + \frac{1}{6}x - 2\right)$

$= \frac{1}{6}\left[6x^2 + 6\left(\frac{1}{6}x\right) - 6(2)\right]$

$= \frac{1}{6}\left[6x^2 + 1x - 12\right]$

$= \frac{1}{6}\left[6x^2 + 9x - 8x - 12\right]$

$= \frac{1}{6}\left[3x(2x+3) - 4(2x+3)\right]$

$p(x) = \frac{1}{6}\left[(2x+3)(3x-4)\right]$

$-12 \times 6x^2$
$-72x^2$
$12 \quad 6$
$9 \quad 8$
$9x \; -8x$

$\alpha + \beta = \left(-1/6\right)$

$\frac{-b}{a} = \frac{-(1/6)}{1} = \left(-1/6\right)$

$\frac{c}{a} = \frac{-2}{1} = -2$

$\alpha \cdot \beta = -2$

$$\frac{1}{6}(2x+3)(3x-4) = 0$$

$$2x+3=0 \Rightarrow x = -3/2 \rightarrow \alpha$$

$$3x-4=0 \Rightarrow x = 4/3 \rightarrow \beta$$

$$\alpha+\beta = -\frac{3}{2} + \frac{4}{3} = \frac{-9+8}{6} = \frac{-1}{6}$$

$$\alpha \cdot \beta = \left(-\frac{\cancel{3}}{\cancel{2}}\right)\left(\frac{\overset{2}{\cancel{4}}}{\cancel{3}}\right) = -2$$

Q. Find the quadratic polynomial for which sum and product of zeroes are – 4 & 3 respectively.

Sum (S) = -4 & prod. (P) = 3

$p(x) = x^2 - Sx + P$

$= x^2 - (-4)x + 3$

$= x^2 + 4x + 3$

Q. Find the quadratic polynomial having zeroes $\sqrt{\frac{3}{2}}$ & $-\sqrt{\frac{3}{2}}$

$\alpha \quad -\alpha$

$Sum\ (S) = \sqrt{\frac{3}{2}} + \left(-\sqrt{\frac{3}{2}}\right) = 0$

$prod.\ (P) = \left(\sqrt{\frac{3}{2}}\right)\left(-\sqrt{\frac{3}{2}}\right) = -\frac{3}{2}$

$x^2 - Sx + P$

$x^2 - 0 - 3/2 \Rightarrow \left(x^2 - 3/2\right)$

Q. The zeroes of the quadratic polynomial $x^2 + kx + k$, $k > 0$,

$a = 1$, $b = k$, $c = k$ (+ve) ; α, β

a. cannot both be positive ✓

b. cannot both be negative

c. are always equal

d. are always unequal

$$\text{Sum} = \alpha + \beta = -\frac{b}{a} = \left(-\frac{k}{1}\right) \rightarrow -ve$$

$$\text{prod.} = \alpha \cdot \beta = \frac{c}{a} = \frac{k}{1} \rightarrow +ve$$

	α	β	
✓ $S \rightarrow (-)$	(−	−) ,	(+ , −)
✓ $P \rightarrow (+)$	(−	−)	(+ +)

Q. If the sum of the zeros of the quadratic polynomial f(t)= $kt^2 + 2t + 3k$ is equal to their product, find the value of k.

$a = k,\ b = 2,\ c = 3k$

$(\alpha + \beta) = \alpha \cdot \beta$

$\frac{-2}{k} = 3$

$k = -2/3$

$\alpha + \beta = \frac{-b}{a} = \frac{-2}{k}$

$\alpha \cdot \beta = \frac{c}{a} = \frac{3k}{k} = 3$

Q. A quadratic polynomial whose zeroes are reciprocal of the zeroes of quadratic polynomial $ax^2 + bx + c, a \neq 0$ & $c \neq 0$ are given by________

α, β

(A) $k(cx^2 + ax + b)$

(B) $k(cx^2 + bx + a)$

(C) $k(cx^2 - bx + a)$

(D) $k(cx^2 + bx - a)$

$f(x) \rightarrow \frac{1}{\alpha}, \frac{1}{\beta}$

$ax^2 + bx + c \rightarrow \alpha + \beta = -\frac{b}{a}$

$\alpha \cdot \beta = \frac{c}{a}$

Sum of Zeroes of U.Q.P $f(x) = \frac{1}{\alpha} + \frac{1}{\beta}$

$$S = \frac{\beta+\alpha}{\alpha\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-b/a}{c/a}$$

$$= \frac{-b}{c} \qquad c \neq 0$$

Prod. of Zeros of $f(x) = \frac{1}{\alpha} \cdot \frac{1}{\beta}$

$$= \frac{1}{\alpha\beta} = \frac{1}{c/a} = \frac{a}{c}$$

$$f(x) = x^2 - S(x) + P$$

$$= x^2 - \left(-\frac{b}{c}\right)x + \frac{a}{c}$$

$$= \frac{c}{c}\left[x^2 + \frac{b}{c}x + \frac{a}{c}\right] \qquad c\left(\frac{a}{c}\right)$$

$$f(x) = \frac{1}{c}\left[cx^2 + bx + a\right]$$

Q. If α & β are the zeroes of the quadratic polynomial $f(x) = x^2 - 1$, find a quadratic polynomial whose zeroes are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$.

$f(x) = x^2 - 1$

$\Rightarrow x^2 - 1 = 0$

$\Rightarrow (x-1)(x+1) = 0$

$x = 1, \quad x = -1$

$\alpha = 1, \quad \beta = -1$

$x^2 - 1^2 = (x-1)(x+1)$

Zeros

$\frac{2\alpha}{\beta} = \frac{2(1)}{-1} = -2$

$\frac{2\beta}{\alpha} = \frac{2(-1)}{1} = -2$

Sum of zeroes (S) $= -2 - 2 = -4$

Product (P) $= (-2)(-2) = 4$

$x^2 - Sx + P$

$x^2 - (-4)x + 4$

$x^2 + 4x + 4$

Q. If α & β are the zeroes of a quadratic polynomial such that $\alpha + \beta = 24$ & $\alpha - \beta = 8$, find a quadratic polynomial whose zeroes are α & β.

$x^2 - Sx + P$ | $\alpha + \beta$, $\alpha \cdot \beta$

$(a+b)^2 = a^2 + b^2 + 2ab$

$(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$ — ①

$(\alpha-\beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$ — ②

$(\alpha+\beta)^2 - (\alpha-\beta)^2 = 4\alpha\beta$

$24^2 - 8^2 = 4\alpha\beta$

$\frac{(24-8)(24+8)}{4} = \alpha\beta$

$\frac{(16)(32)}{4} = \alpha\beta$

$\alpha\beta = 16 \times 8$

$= 128$

$x^2 - Sx + P$

$x^2 - 24x + 128$

Homework questions

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes & the coefficient.

   p(s) = $4s^2 - 4s + 1$

1. If α & β are the zeroes of the quadratic polynomial p(y) $= 5y^2 - 7y + 1$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

2. If a & b are the zeroes of the quadratic polynomial f(x) = $x^2 - 2x + 3$, find a polynomial whose roots are $a + 2, b + 2$.