Deeksha

# ABHYAS KCET 2024


# FREE TEST SERIES

## MOCK TEST 3 PDF

## SOLUTION

Deeksha - CET

| Subject | Topic | |
|---|---|---|
| C + M + P | Complete Syllabus | |

**Max. Marks:** **180** **Duration:** **3 Hours**

1. **This paper consists of 180 questions with 3 parts of Chemistry, Mathematics and Physics**

   - **Chemistry:** *(Q. No. 1 to 60) Multiple Choice Questions with one correct answer. A correct answer carries 1 Mark. No Negative marks.*
   - **Mathematics:** *(Q. No. 61 to 120) Multiple Choice Questions with one correct answer. A correct answer carries 1 Mark. No Negative marks.*
   - **Physics:** *(Q. No. 121 to 180) Multiple Choice Questions with one correct answer. A correct answer carries 1 Mark. No Negative marks.*

2. **The OMR sheet for 200 questions is to be used**

3. **Use of calculators and log tables is prohibited**

4. **Darken the appropriate bubble using a pen in the OMR sheet provided to you. Once entered, the answer cannot be changed. Any corrections or modifications will automatically draw a penalty of 1 mark**

5. **No clarification will be entertained during the examination. Doubts in the paper can be reported to the coordinator after the exam**

6. **If the details in the OMR Sheet are not filled, If the OMR sheet is mutilated, torn, white Ink used, the circles filled and scratched, then the OMR sheet will not be graded**

   **All the best!!**

## Useful Data

**At. Wt.:**

$N=14;\ O=16;\ H=1;\ S=32;\ Cl=35.5;\ Mn=55;\ Na=23;\ C=12;\ Ag=108;\ K=39;\ Fe=56;\ Pb=207$

**Physical Constants:**

$h=6.626\times10^{-34}\,\mathrm{Js}$, $\mathrm{N_a}=6.022\times10^{23}\,\mathrm{mol^{-1}}$, $\mathrm{c}=2.998\times10^{8}\,\mathrm{m\,s^{-1}}$, $\mathrm{m_e}=9.1\times10^{-31}\,\mathrm{kg}$, $R=8.314\ \mathrm{J\,mol^{-1}\ K^{-1}}$

## Chemistry

**Multiple Choice Questions with one correct answer. A correct answer carries 1 mark. No negative mark. 60 x 1 = 60**

1. The pair of species having same percentage of carbon is

   Options:

   (a) $C_6H_{12}O_6$ and $C_{12}H_{22}O_{11}$ (b) $CH_3COOH$ and $C_2H_5OH$

   (c) $CH_3COOH$ and $C_6H_{12}O_6$ (d) $HCOOH$ and $CH_3COOH$

   Sol: $CH_3COOH$ and $C_6H_{12}O_6$ have same empirical formula.

   Hence have same percentage of carbon.

   Ans: (c)

2. Which of the following sets of quantum numbers represents an impossible arrangement?

   Options:

   | | $n$ | $\ell$ | $m$ | $s$ |
   |---|---|---|---|---|
   | (a) | 2 | 2 | $-2$ | $+\frac{1}{2}$ |
   | (b) | 4 | 0 | 0 | $-\frac{1}{2}$ |
   | (c) | 5 | 2 | 0 | $+\frac{1}{2}$ |
   | (d) | 3 | 3 | $+2$ | $+\frac{1}{2}$ |

   Sol: When $n = 3$ $\ell$ cannot be 3

   Ans: (d)

3. Which of the following statements is correct

   Options:

   (a) Ionization enthalpy of $Mg$ is less than that of $Na$ and $Al$

   (b) The atomic radius of $F$ is more than that of $O$

   (c) Negative electron gain enthalpy of $F$ is less than that of $O$

   (d) Among $Be, B$ and $C$, $B$ has lowest ionization enthalpy

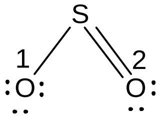
   Sol: $IF_c$ of $Be \to 900\,\text{kJ/mol}$ $EA$ of $F = 328\,\text{kJ/mol}$

   $IF_c$, of $B \to 800\,\text{kJ/mol}$ $EA$ of $o = -141\,\text{kJ/mol}$

   $IF_c$, of $C \to 1090\,\text{kJ/mol}$

   Ans: (d)

4. Formal charge on two $O$ atoms in

   Options:

   (a) $-1, +1$ (b) $-1, 0$ (c) $0, +1$ (d) $-1, -1$

Sol: $FC = V - L - \frac{1}{2}S$

$FC_{01} = 6 - 6 - \frac{1}{2}(2)$

$= -1$

$FC_{O2} = 6 - 4 - \frac{1}{2}(4)$

$= 0$

Ans: (b)

5. A gaseous mixture was prepared by taking equal mole of $CO$ and $N_2$ of the total pressure of the mixture was found to be $1\,\text{atm}$, the partial pressure of nitrogen $(N_2)$ in the mixture is

Options:

(a) $0.5\,\text{atm}$ (b) $0.8\,\text{atm}$ (c) $0.9\,\text{atm}$ (d) $1\,\text{atm}$

Sol: Let number of moles of $CO$ and $N_2 = x$

$X_{co} = \frac{x}{2x} = \frac{1}{2}$ and $X_{N_2} = \frac{1}{2}$

$\therefore P_{N_2} = X_{N_2} \times P_{total} = \frac{1}{2} \times 1 = 0.5$

Ans: (a)

6. For which of the following reaction, $\Delta S$ is not positive?

Options:

(a) $I_2(s) \rightarrow I_2(g)$ (b) $CuO(s) + H_2(g) \rightarrow Cu(g) + H_2O(l)$

(c) $2O_3(g) \rightarrow 3O_2(g)$ (d) $2Ag_2O(s) \rightarrow 4Ag + O_2(g)$

Sol: $CuO(s) + H_2(g) \rightarrow Cu(s) + H_2O(l)$

$\Delta n = 0 - 1 = -1$

$\therefore \Delta S$ is negative.

Ans: (d)

7. The heat of combustion of carbon to $CO_2$ is $-393.5\,\text{kJ/mol}$

The heat released for the formation of $22g$ of $CO_2$ from carbon and oxygen is

Options:

(a) $-393.5\,\text{kJ/mol}$ (b) $-39.3\,\text{kJ/mol}$ (c) $-19.6\,\text{kJ/mol}$ (d) $-196.75\,\text{kJ/mol}$

Sol: Heat released for forming

$44g$ of $CO_2 = -393.5\,\text{kJ}$

$\therefore$ Heat released for forming

$22g$ of $CO_2 = \frac{-393.5 \times 22}{44} = -196.75\,\text{kJ/mol}$

Ans: (d)

8. The precipitate of Calcium fluoride $(CaF_2)$ with $K_{sp} = 1.7\times10^{-10}$ is obtained when equal volumes of the following are mixed. The mixture which gives precipitate is

Options:

(a) $10^{-4} MCa^{2+}$ and $10^{-4} MF^{-}$
(b) $10^{-2} MCa^{2+}$ and $10^{-3} MF^{-}$
(c) $10^{-5} MCa^{2+}$ and $10^{-3} MF^{-}$
(d) $10^{-5} MCa^{2+}$ and $10^{-5} MF^{-}$

Sol: $Ksp$ of $CaF_2 = \left[Ca^{2+}\right]\left[F^{-}\right]^2 = 1.7\times10^{-10}$

$IP$ of $CaF_2 = 10^{-2}\times\left(10^{-3}\right)^2 = 10^{-8}$ for

option (b)

Precipitation happens when

$IP > K_{SP}$

Ans: (b)

9. $Ka_1$, $Ka_2$ and $Ka_3$ are respective constants for the following reactions.

$H_2S \rightleftharpoons H^+ + HS^- \quad Ka_1$

$HS^- \rightleftharpoons H^+ + S^{2-} \quad Ka_2$

$H_2S \rightleftharpoons 2H^+ + S^{2-} \quad Ka_3$

The correct relationship between $Ka_1, Ka_2$ and $Ka_3$ is

Options:

(a) $Ka_3 = Ka_1 \times Ka_2$
(b) $Ka_3 = Ka_1 + Ka_2$
(c) $Ka_3 = Ka_1 - Ka_1$
(d) $Ka_3 = \frac{Ka_1}{Ka_2}$

Sol: $Ka_3 = Ka_1 \times Ka_2$

Ans: (a)

10. $3C10^{-}(aq) \rightarrow C10_3^{-} + 2C1^{-}$ is an example of

Options:

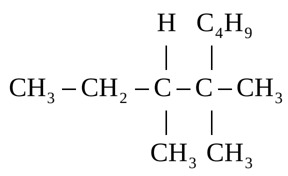
(a) Oxidation reaction
(b) Reduction reaction
(c) Disproportionation reaction
(d) Displacement reaction

Sol: $3\overset{+1}{C10^{-}}(aq) \rightarrow \overset{+5}{C10_3^{-}} + 2\overset{-1}{C1^{-}}$

Chlorine is undergoing oxidation as well as reduction.

Ans: (c)

11. The IUPAC name of

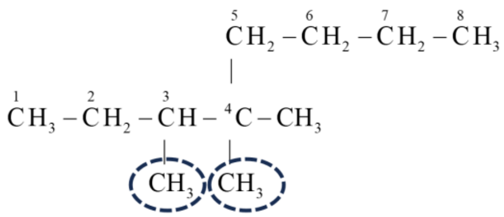


Options:

(a) 3,4,4- Trimethylheptane (b) 3,4,4- Trimethyloctane

(c) 2-Butyl-2-methyl-3-ethylbutane (d) 2-Ethyl-3,3-dimethylheptane

Sol:



3,4,4-Trimethyloctane

Ans: (b)

12. The number of atoms in 52 u of *He* are

Options:

(a) 13 (b) $13\times 6.022\times 10^{23}$ (c) 52 (d) $4\times 6.022\times 10^{23}$

Sol: At. Mass of $He = 4$ u

4 u of $He$ has atoms $=1$

$\therefore$ 52 u of $He$ has atoms $=\frac{52}{4}=13$

Ans: (a)

13. The total number of isomeric alcohols with the molecular formula $C_4H_9OH$ is:

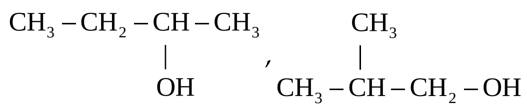
Options:

(a) 3 (b) 4 (c) 5 (d) 2

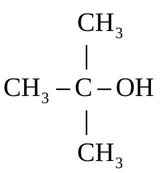
Sol: $C_4H_9OH$ has four isomeric alcohols.

$CH_3-CH_2-CH_2-CH_2-OH$,

Butan-1-ol



Butan-2-ol , 2-Methylpropan-1-ol

and 

Ans: (b)

14. In Duma's method $0.03g$ of an organic compound gave $41.9\,\text{ml}$ of nitrogen at STP. The percentage of $N$ is

Options:

(a) 29.46% (b) 25.2% (c) 17.37% (d) 39.2%

Sol: $\%\text{ of } N = \frac{28}{22,400}\times\frac{V_{STP}}{W_{o,c}}\times 100$

$= \frac{28}{22,400}\times\frac{41.9}{0.3}\times 100 = 17.37$

Ans: (c)

15. Which of the following is most reactive towards sodium?

Options:

(a) $CH_3-C=CH$ (b) $CH_3-C=C-CH_3$

(c) $CH_3-CH_2-C\equiv CH$ (d) $CH\equiv CH$

Sol: $CH\equiv CH$ is most acidic and hence most reactive towards sodium.

Ans: (d)

16. In the following sequence of reaction, the end product is

$$CaC_2 \xrightarrow{H_2O} A \xrightarrow{Hg^{2+}/H_2SO_4} B \xrightarrow{[O]} C \xrightarrow{Ca(OH)_2} \xrightarrow{Heat} E$$

Options:

(a) Acetaldehyde (b) Formaldehyde (c) Acetic acid (d) Acetone

Sol: $CaC_2 \xrightarrow{H_2O} CH\equiv CH \xrightarrow{Hg^{2+}/H_2SO} CH_3CHO \xrightarrow{[O]} CH_3COOH \xrightarrow{Ca(OH)_2}$

$$CH_3-\overset{\overset{O}{\|}}{C}-CH_3 \xleftarrow{\text{heat}} (CH_3COO)_2\,Ca$$

Ans: (d)

17. Which of the following is ***not*** a conductor of electricity?

Options:

(a) Solid NaCl (b) Cu (c) Fused NaCl (d) Brine solution

Sol: Electrolytes conduct electricity only in aqueous or molten state but not in solid state, so, solid NaCl is not a conductor.

Ans: (a)

18. For an ideal binary liquid mixture

Options:

(a) $\Delta H_{(mix)}=0;\Delta S_{(mix)}<0$ (b) $\Delta S_{(mix)}>0;\Delta G_{(mix)}<0$

(c) $\Delta S_{(mix)}=0;\Delta G_{(mix)}=0$ (d) $\Delta V_{(mix)}=0;\Delta G_{(mix)}>0$

Sol: For an ideal binary mixture,

$\Delta H_{mix}=0, \Delta V_{mix}=0.\ \Delta G_{mix}<0$ and $\Delta S_{mix}>0$.

Ans: (b)

19. The molal elevation constant is the ratio of elevation in boiling point to

Options:

(a) Molarity (b) Boiling point of pure liquid

(c) Mole fraction of solute (d) Molality

Sol: $\Delta T_b = K_p m$

$K_b = \frac{\Delta T_b}{m}$ where $'m'$ is the molality of the solution.

Ans: (d)

20. A plant cell shrinks when placed in

Options:

(a) Water (b) Hypotonic solution

(c) Isotonic solution (d) Hypertonic solution

Sol: A plant cell shrinks when placed in hypertonic solution

Ans: (d)

21. Two moles of a non-volatile solute are dissolved in $5$ moles of water. The vapour pressure of the solute relative to that of water is

Options:

(a) $\frac{2}{5}$ (b) $\frac{2}{7}$ (c) $\frac{4}{7}$ (d) $\frac{5}{7}$

Sol: $\frac{P_o - P_S}{P_o} = X_2 = \frac{n_2}{n_1 + n_2} = \frac{2}{2+5} = \frac{2}{7}$

$1 - \frac{P_s}{P_o} = \frac{2}{7}$ or $\frac{P_s}{P_o} = 1 - \frac{2}{7} = \frac{5}{7}$

Ans: (d)

22. In the Laclanche dry cell, anode is

Options:

(a) Graphite rod (b) Carbon (c) Zinc container (d) $MnO_2 + C$

Sol: Anode – Zinc container

Ans: (c)

23. The emf of the cell at $25°C$

$Cu / Cu^{2+}(0.01M) || Ag^{+}(0.1\text{M}) / Ag$ is

$$\left(\text{Given } E^{\circ}_{cell \frac{2t}{Cu}} = 0.34V \text{ and } E^{\circ}_{\frac{Ag^+}{Ag}} = 0.80V\right)$$

Options:

(a) $0.46V$ (b) $1.14V$ (c) $0.43V$ (d) $1.29V$

Sol: $E_{cell} = E^{\circ}_{cell} - \frac{0.0591}{n} \log \frac{Cu^{2+}}{\left(Ag^+\right)^2}$

$$=(0.80-0.34)-\frac{0.0591}{2}\log 1=0.46V$$

Ans: (a)

24. The quantity of electricity needed to separate the electrolyte of 1M solution of $ZnSO_4, AlCl_3$ and $AgNO_3$ completely is in the ratio of

Options:

(a) 2 : 3 : 1 (b) 2 : 1 : 1 (c) 2 : 1 : 3 (d) 2 : 2 : 1

Sol: $Zn^{2+}(1M)+2e^{-}\rightarrow Zn$ charge required $2F$

$Al^{3+}(1M)+3e^{-}\rightarrow Al$ charge required $3F$

$Ag^{+}(1M)+e^{-}\rightarrow Ag$ charge required $F$

Ans: (a)

25. What is the activation energy for a reaction if its rate doubles when the temperature is raised from 300 k to 310 k ?

Options:

(a) $535\,\text{kJ mol}^{-1}$ (b) $5350\,\text{kJ mol}^{-1}$ (c) $53.5\,\text{kJ mol}^{-1}$ (d) $5.35\,\text{kJ mol}^{-1}$

Sol: $\log\frac{K_2}{K_1}=\frac{E_a}{2.303R}\left[\frac{T_2-T_1}{T_2T_1}\right]$

$$\log 2=\frac{E_a}{2.303\times 8.314}\left[\frac{10}{300\times 310}\right]$$

$E_a=0.301\times 19.14\times 30\times 310=53,578\text{J}=53.5\text{kJ/mol}$

Ans: (c)

26. The time required for 100 percent completion of a zero order reaction is:

Options:

(a) $\frac{2k}{a}$ (b) $\frac{a}{2k}$ (c) $\frac{a}{k}$ (d) $ak$

Sol: For zero order reaction

$$k=\frac{[A]_0-[A]}{t}$$

For 100 percent completion

$[A]=\text{zero}$

$$k=\frac{[A]_0}{t}\text{ or }t=\frac{[A]_0}{k}$$

$t=\frac{a}{k}$, where $a$ is initial concentration.

Ans: (c)

27. A first order reaction is half completed in 45 minutes. How long does it need for 99.9% of the reaction to be completed?

Options:

(a) 10 hr (b) 20 hr (c) 5 hr (d) 7.5 hr

Sol: $k = \frac{0.693}{45 \text{ min}}; t = \frac{2.303}{k} \log \frac{100}{0.1}$

$t = \frac{2.303 \times (45) \times 3}{0.693 \times 60} \text{hr} = 7.5 \text{ hr}$

Ans: (d)

28. Aluminium is more reactive than iron but aluminium is less easily corroded than iron because

Options:

(a) aluminium is a noble metal

(b) oxygen forms a protective oxide layer on aluminium surface

(c) iron undergoes reaction easily with water

(d) iron form both divalent and trivalent ions.

Sol: Protective oxide layer is formed on the surface.

Ans: (b)

29. The quantity of electricity required to liberate $112 \text{ cm}^3$ of hydrogen at STP from acidulated water is

Options:

(a) 965 C (b) 1 Faraday (c) 0.1F (d) 96500 C

Sol:

$$2H^+ + 2e^- \rightarrow H_2$$
$$2F \quad 1 \text{ mole } (22400 \text{ cm}^3 \text{ at STP})$$

$22400 \text{ cm}^3$ of $H_2$ at STP is produced by passing electricity $= 2 \times 96500$ C

$\therefore 112 \text{ cm}^3$ of $H_2$ at STP is produced by passing electricity $= \frac{2 \times 96500}{22400} \times 112 = 965$ C

Ans: (a)

30. Which of the following has the maximum number of unpaired electrons?

Options:

(a) $Mg^{2+}$ (b) $Ti^{3+}$ (c) $V^{3+}$ (d) $Fe^{2+}$

Sol: $Mg^{2+}$ has electronic configuration

$1s^2 2s^2 2p^6$; no unpaired electrons

${}_{22}Ti^{3+}$ has electronic configuration $[Ar]3d^1$;one unpaired electron

${}_{23}V^{3+}$ has electronic configuration $[Ar]3d^2$;two unpaired electron

${}_{26}Fe^{2+}$ has electronic configuration $[Ar]3d^6$; four unpaired electrons.

Ans: (d)

31. Which of the following statement is wrong regarding Lanthanoids?

Options:

(a) Ln(III) compounds are generally colourless

(b) Ln(III) compounds are predominantly ionic in character.

(c) The ionic size of Ln(III) ions decreases with increasing atomic number

(d) Ln(III) hydroxides are mainly basic in nature.

Sol: Many of the trivalent ions of Lanthanoids are coloured, both in the solid state as well as in solution. This is due to partly filled $f-$subshell and $f-f$ transitions due to absorption of light from visible region.

Ans: (a)

32. In the coordination compound $K_4[Ni(CN)_4]$, the oxidation state of nickel is

Options:

(a) 0 (b) +1 (c) +2 (d) -1

Sol: Let $x$ be the oxidation state of nickel

$x+(+1\times4)+(-1\times4)=0$ or $x=0$

Ans: (a)

33. The formula of pentaaquanitratochromium(III) nitrate is

Options:

(a) $[Cr(H_2O)_6](NO_3)_3$ (b) $[Cr(H_2O)_5NO_3](NO_3)_2$

(c) $[Cr(H_2O)_6](NO_2)_2$ (d) $[Cr(H_2O)_5NO_2]NO_3$

Sol: $[Cr(H_2O)_5NO_3](NO_3)_2$ is the correct answer

Ans: (b)

34. Among the following the square planar geometry is for

Options:

(a) $XeF_3$ (b) $XeF_4$ (c) $XeF_2$ (d) $XeO_3$

Sol: $XeF_4$ has square planar geometry

Ans: (b)

35. The number of moles of $KMnO_4$ that will be needed to react with one mole of sulphite ion in acidic solution is

Options:

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) 1

Sol: $2MnO_4^- + 5SO_3^{2-} + 6H^+ \rightarrow 2Mn^{2+} + 5SO_4^{2-} + 3H_2O$

Ans: (a)

36. Which of the following pairs has the same size?

Options:

(a) $Zr^{4+}, Hf^{4+}$ (b) $Zn^{2+}, Hf^{4+}$ (c) $Fe^{2+}, Ni^{2+}$ (d) $Zr^{4+}, Ti^{4+}$

Sol: $Zr^{4+}, Hf^{4+}$ show similar size because of lanthanide contraction.

Ans: (a)

37. The ion sowing a magnetic moment of 2.83BM among the following is

Options:

(a) $Ti^{3+}$ (b) $Ni^{2+}$ (c) $Cr^{3+}$ (d) $Mn^{2+}$

Sol: 2.83 BM magnetic moment is shown by ion with 2 unpaired electrons i.e. $Ni^{2+}$, $\mu = \sqrt{n(n+2)}$

Ans: (b)

38. The crystal field splitting energy for octahedral $(\Delta_o)$ and tetrahedral $(\Delta t)$ complexes is related as

Options:

(a) $\Delta_t = \frac{1}{2}\Delta_o$ (b) $\Delta_t = \frac{4}{9}\Delta_o$ (c) $\Delta_t = \frac{3}{5}\Delta_o$ (d) $\Delta_t = \frac{2}{5}\Delta_o$

Sol: $\Delta_t = \frac{4}{9}\Delta_o$

Ans: (b)

39. Which of the following statements is not correct?

Options:

(a) $[FeF_6]^{3-}$ has five unpaired electrons

(b) $[Co(NH_3)_3Cl_3]$ is an non-conductor

(c) Tetrahedral complexes do not show geometrical isomerism

(d) In $CN$ group, bonding occurs through $N$

Sol: In $CN$ group, bonding occurs through $N$

Ans: (d)

40. Which of the following is a outer orbital complex?

Options:

(a) $[Fe(CN)_6]^{4-}$ (b) $[CoF_6]^{3-}$ (c) $[Co(NH_3)_6]^{3+}$ (d) $[Fe(cn)_3]^{2+}$

Sol: $[CoF_6]^{3-}$ is an outer orbital complex

Ans: (b)

41. The addition of a catalyst during a chemical reaction alters which of the following quantities?

Options:

(a) Entropy (b) Internal energy (c) Enthalpy (d) Activation energy

Sol: Catalyst changes the path of the reaction and reduces the activation energy.

Ans: (d)

42. The rate for the first order reaction is $0.0069\,mol\,L^{-1}\,min^{-1}$ and the initial concentration is $0.2\,mol\,L^{-1}$. The half-life period is

Options:

(a) 10mins (b) 20mins (c) 15 min (d) 7 min

Sol: Rate = $K[R]$

$$\therefore K = \frac{0.0069}{0.2} = 0.0345$$

$$t_{\frac{1}{2}} = \frac{0.693}{k} = \frac{0.693}{0.0345} = 20.08\text{mins}$$

Ans: (b)

43. Ethyl isocyanide is prepare by the reaction between

Options:

(a) $C_2H_5Br$ and $KCN(alc)$ (b) $C_2H_5Br$ and $AgCN(alc)$

(c) $C_2H_5Br$ and $HCN$ (d) $C_2H_5Br$ and ammonia

Sol: $C_2H_5Br + AgCN(acl) \rightarrow C_2H_5NC + AgBr$

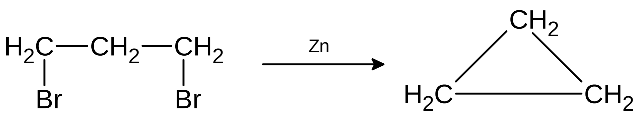
Ans: (b)

44. $1,3-$ Dibromopropane reacts with metallic zinc to form

Options:

(a) Propene (b) Propane (c) Hexane (d) Cyclopropane

Sol:




Ans: (d)

45. Which of the following is most reactive towards $S_N1$ reaction?

Options:

(a) Methyl bromide (b) Tertiary butyl bromide

(c) Secondary butyl bromide (d) Ethyl bromide

Sol: Methyl bromide

Ans: (a)

46. An alkene $CH_3CH = CH_2$ is treated with $B_2H_6$ in presence of $H_2O_2$. The final product formed is

Options:

(a) $CH_3CH_2CHO$ (b) $CH_3CH(OH)CH_3$ (c) $CH_3CH_2CH_2OH$ (d) $(CH_3CH_2CH_2)_3B$

Sol: $CH_3CH = CH_2 + (BH_3)_2 \rightarrow CH_3CH_2CH_2BH_2 \xrightarrow{CH_3CH=CH_2} (CH_3CH_2CH_2)BH$

$\xrightarrow{CH_3CH=CH_2} (CH_3CH_2CH_2)_2B \xrightarrow[3H_2O_2/OH]{H_2O} 3CH_3CH_2CH_2OH + B(OH)_3$

Ans: (c)

47. Acid catalysed dehydration of $t-$butanol is faster than that of $n-$butanol because

Options:

(a) tertiary carbocation is more stable than primary carbocation

(b) primary carbocation is more stable than tertiary carbocation

(c) $t-$butanol has higher boiling point

(d) rearrangement takes place during dehydration of $t-$butanol

Sol: Teritiary carbocation is more stable than $1°$, hence the dehydration in $3°$ alcohol proceeds faster than $1°$ alcohol.

Ans: (a)

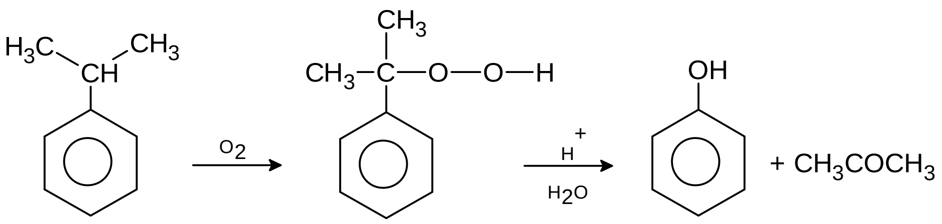
48. Cumene on reaction with oxygen followed by hydrolysis gives

Options:

(a) $CH_4OH$ and $C_6H_5COCH_3$ (b) $C_6H_5OH$ and $(CH_3)_2O$

(c) $C_6H_5OCH_3$ and $CH_3OH$ (d) $C_6H_5OH$ and $CH_3COCH_3$

Sol:




Ans: (d)

49. Anisole on reaction with chloromethane in presence of anhydrous $AlCl_3$ gives

Options:

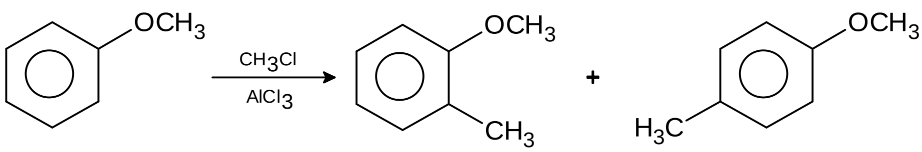
(a) $o-$ methylanisole and $p-$ methoxyanisole

(b) $p-$ methylanisole and $p-$ methoxyanisole

(c) $o-$ methylanisole and $p-$ methoxyanisole

(d) $o-$ methoxyacetophenone and $p-$ methoxyacetophenone

Sol:




Ans: (c)

50. The most acidic among the following

Options:

(a) Phenol (b) $p-$Cresol (c) $p-$Nitrophenol (d) $2,4-$Dinitrophenol

Sol: $2,4-$ Dinitrophenol

Ans: (d)

51. Which of the following compound does not react with $NaHSO_3$ ?

Options:

(a) $HCHO$ (b) $C_6H_5COCH_3$ (c) $CH_3COCH_3$ (d) $CH_3CHO$

Sol: $C_6H_5COCH_3$ does not react with $NaHSO_3$ because of steric hindrance

Ans: (b)

52. A compound $(X)$ with a molecular formula $C_5H_{10}O$ gives a positive $2,4-DNP$ test but a negative Tollen's test. On oxidation it gives carboxylic acid $(Y)$ with a molecular formula $C_3H_6O_2$. Potassium salt of $(Y)$ undergoes Kolbe's reaction to give a hydrocarbon $(2)$. $X$. $Y$ and $Z$ respectively are

Options:

(a) Pentan $-3-$ one, propanoicacid, butane

(b) Pentanol, pentanoic acid, octane

(c) $2-$ Methylbutanone, butanoic acid, hexane

(d) $2,2-$ dimethylpropanone, propanoic acid, hexane

Sol: $CH_3-CH_2-\overset{\overset{O}{\|}}{C}-CH_2-CH_3 \xrightarrow{[O]} CH_3CH_2COOH + CH_3COOH \xrightarrow{KOH} CH_3CH_2-COOK$

$\xrightarrow{\text{Kolbe's electrolysis}} C_4H_{10} + 2CO_2$

Ans: (a)

53. Complete the missing links $(X),(Y)$ and $(Z)$ by making an appropriate choice

$CH_3COOH \xrightarrow{PBr_3/Br_3} X \xrightarrow{KCN} Y \xrightarrow{H_3O} Z$

Options:

| | $X$ | $Y$ | $Z$ |
|---|---|---|---|
| (a) | $CH_3COBr$ | $CH_3COCN$ | $CH_3COOH$ |
| (b) | $BrCH_2COOH$ | $\underset{\underset{CN}{\mid}}{CH_2}-COOH$ | $HOOC-CH_2-COOH$ |
| (c) | $BrCH_2COOH$ | $CH_2(CN)COOH$ | $COOH-COOH$ |
| (d) | $Br_2CH-COOH$ | $Br_2C(CN)COOH$ | $CH_3CH_2COOH$ |

Sol: $CH_3COOH \xrightarrow{PBr/Br_2} \underset{\underset{Br}{\mid}}{CH_2}-COOH \xrightarrow{KCN} \underset{\underset{CN}{\mid}}{CH_2}-COOH \xrightarrow{H_3O^+} \underset{\underset{COOH}{\mid}}{CH_2}-COOH$

Ans: (b)

54. Which of the following will form isocyanide on reaction with $CHCl_3$ and $KOH$ ?

Options:

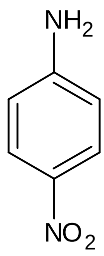
(a) $C_6H_5NHCH_3$ (b) $CH_3C_6H_4NH_2$ (c) $C_6H_5NHC_4H_9$ (d) $C_6H_5N(C_2H_5)_2$

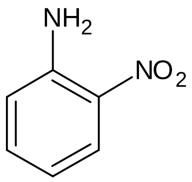
Sol: Only primary amines undergo carbylamine reaction

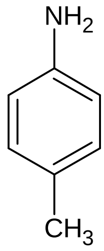
Ans: (b)

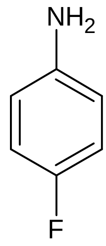
55. The most basic amine among the following is

Options:

(a) 

(b) 

(c) 

(d) 

Sol: Electron releasing groups increases basicity of amines

Ans: (c)

56. On oxidation with a mild oxidising agent like $Br_2 / H_2O$ the glucose is oxidised to

Options:

(a) Saccharic acid (b) Glucaric acid (c) Gluconic acid (d) Valeric acid

Sol: Glucose $\xrightarrow[\text{Oxidation}]{Br_2/H_2O}$ gluconic acid

Ans: (c)

57. Which of the following vitamins is water soluble?

Options:

(a) Vitamin E (b) Vitamin D (c) Riboflavin (d) Retinol

Sol: Riboflavin

Ans: (c)

58. In fibrous proteins polypeptide chains are held together by

Options:

(a) Vander Waal's forces (b) Electrostatic forces of attraction

(c) Hydrogen bonds (d) Covalent bonds

Sol: Polypeptide chain in fibrous proteins are held together by hydrogen bonds

Ans: (c)

59. Hofmann's bromamide reaction is to convert

Options:

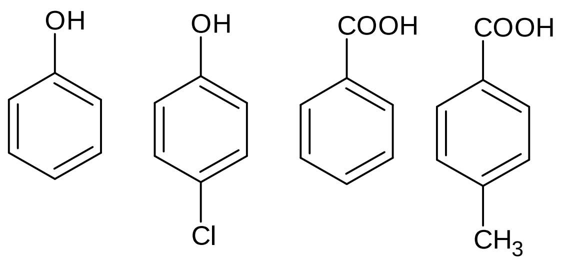
(a) alcohol to acid (b) acid to alcohol (c) amine to amide (d) amide to amine

Sol:

$$\underset{\text{Amide}}{R-\overset{\overset{\displaystyle O}{\|}}{C}-NH_2} \xrightarrow[\Delta]{Br_2+KOH} \underset{\text{Amine}}{R-NH_2}$$

Ans: (d)

60. The correct acidity order of the following is



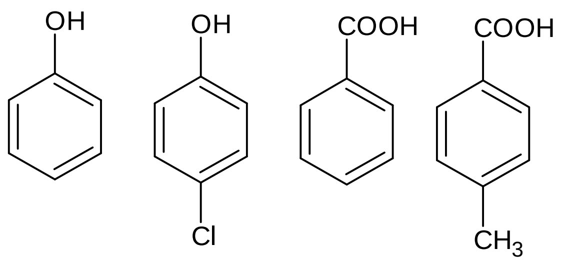

I II III IV

Options:

(a) III > IV > II > I (b) IV > III > I > II (c) III > II > I > IV (d) II > III > IV > I

Sol: Phenols are weaker acids than carboxylic acids so III and IV are stronger acids that I and II. Now III is stronger acids than IV because electron releasing $-CH_3$ group in IV lowers the acidity. II is stronger acid than I because of -I effect of -Cl. Thus decreasing order of acidic strength:

III > IV > II > I




I II III IV

$pK_a = 9.98$ $pK_a = 9.38$ $pK_a = 4.17$ $pK_a = 4.37$

Ans: (a)

## Mathematics

**Multiple Choice Questions with one correct answer. A correct answer carries 1 mark. No negative mark. 60 x 1 = 60**

61. In the set $A=\{1,2,3,4,5\}$, a relation $R$ is defined by $R=\{x,y):x,\ y\in A,\ x<y\}$. Then $R$ is

Options:

(a) Reflexive (b) Symmetric (c) Transitive (d) None of these

Sol: The relation '<" is only a transitive relation.

Ans: (c)

62. The domain of the function $f(x)=\sqrt{\left(2-2x-x^2\right)}$ is

Options:

(a) $-1\le x\le\sqrt{3}$ (b) $-1-\sqrt{3}\le x\le -1+\sqrt{3}$ (c) $-2\le x\le 2$ (d) None of these

Sol: $f(x)=\sqrt{2-2x-x^2}$ is defined for all $x$ for which, $2-2x-x^2>0$

i.e., for which $x^2+2x-2<0$

Consider $x^2+2x-2=0$

$$\Rightarrow\ x=\frac{-2\pm\sqrt{4+8}}{2}$$

$$\Rightarrow\ x=-1\pm\sqrt{3}$$

Thus $x^2+2x-2<0,$ for $-1-\sqrt{3}\le x\le -1+\sqrt{3}$

$\left[ax^2+bx+c\le 0 \text{ for } \alpha\le x\le\beta, \text{where } \alpha \text{ and } \beta \text{ are the roots of } ax^2+bx+c=0,\ \alpha<\beta\right]$

Ans: (b)

63. The mapping $f:R^+\to R$ defined by $f(x)=\log_{10}x$, (where $R^+$ is the set of all positive real numbers) is

Options:

(a) Only one-one mapping (b) Only onto mapping

(c) Both one-one and onto (d) None of these

Sol: $f(x)=\log_{10}x$ is defined for all $x>0.$

Further, it is both one-one and onto function.

Ans: (c)

64. If $g(x)=1+\sqrt{x}$ and $f(g(x))=3+2\sqrt{x}+x$ then $f(x)=$

Options:

(a) $1+2x^2$ (b) $2+x^2$ (c) $1+x$ (d) $2+x$

Sol: $f(g(x))=3+2\sqrt{x}+x=\left[\left(\sqrt{x}\right)^2+2\sqrt{x}+1\right]+2=\left(\sqrt{x}+1\right)^2+2=\left[g(x)\right]^2+2$

$$\Rightarrow f(x)=x^2+2$$

Ans: (b)

65. Let, $f: R \to R$ be defined by $f(x) = \begin{cases} 2x & x > 3 \\ x^2 & 1 < x \le 3 \\ 3x & x \le 1 \end{cases}$ *Then* $f(-1) + f(2) + f(4) =$

Options:

(a) 9 (b) 14 (c) 5 (d) None of these

Sol: Now, $f(-1) = 3(-1) = -3 \quad (\therefore -1 < 1)$

$f(2) = 2^2 = 4 \quad (\because -1 < 2 < 3)$

$f(4) = 2(4) = 8 \quad (\therefore 4 > 3)$

$f(-1) + f(2) + f(4) = -3 + 4 + 8 = 9$

Ans: (a)

66. If one root of the equation $5x^2 + 13x + k = 0$ is reciprocal of other, then the value of $k$ is

Options:

(a) 0 (b) 5 (c) $\frac{1}{6}$ (d) 6

Sol: Let $\alpha, \frac{1}{\alpha}$ be the roots

$\Rightarrow \alpha + \frac{1}{\alpha} = -\frac{13}{5}$ and $\alpha.\frac{1}{\alpha} = \frac{k}{5} \Rightarrow \frac{k}{5} = 1 \Rightarrow k = 5$

Ans: (b)

67. The number of ways in which ten candidates $A_1, A_2, .........., A_{10}$ be ranked, if $A_1$ is always above $A_2$ is

Options:

(a) $2 \times 8!$ (b) $9!$ (c) $10!$ (d) $5 \times 9!$

Sol: Ten candidates can be ranked in $10!$ ways. In half of there $A_2$ and in another half $A_2$ is ranked above $A_1$

$\therefore$ required number $= \frac{1}{2} \times 10! = 5 \times 9!$

Ans: (d)

68. If $A = \begin{pmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{pmatrix}$ and $B = \begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix}$

Then $AB =$

Options:

(a) $B$

(b) $A$

(c) $O$, where $O$ is null matrix

(d) $I_3$, where $I_3$ is unit matrix of order 3

Sol: $A\cdot B=\begin{pmatrix}0 & c & -b\\ -c & 0 & a\\ b & -a & 0\end{pmatrix}\begin{pmatrix}a^2 & ab & ac\\ ab & b^2 & bc\\ ac & bc & c^2\end{pmatrix}=\begin{pmatrix}abc-abc & b^2c-b^2c & bc^2-bc^2\\ -a^2c+a^2c & -abc+abc & -ac^2+ac^2\\ a^2b-a^2b & ab^2-ab^2 & abc-abc\end{pmatrix}=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}=O$

Ans: (c)

69. If $A=\begin{pmatrix}x & 1\\ 0 & x\end{pmatrix}$, then $A^n=$

Options:

(a) $\begin{pmatrix}x^n & nx^{n-1}\\ 0 & x^n\end{pmatrix}$ (b) $\begin{pmatrix}nx^{n-1} & x^n\\ 0 & x^n\end{pmatrix}$ (c) $\begin{pmatrix}x^n & 0\\ nx^{n-1} & x^n\end{pmatrix}$ (d) $\begin{pmatrix}x^n & x^n\\ 0 & x^{n-1}\end{pmatrix}$

Sol: Consider, $A^2=\begin{pmatrix}x & 1\\ 0 & x\end{pmatrix}\begin{pmatrix}x & 1\\ 0 & x\end{pmatrix}=\begin{pmatrix}x^2 & 2x\\ 0 & x^2\end{pmatrix}=\begin{pmatrix}x^2 & 2x^{2-1}\\ 0 & x^2\end{pmatrix}$

$A^3=\begin{pmatrix}x & 1\\ 0 & x\end{pmatrix}\begin{pmatrix}x^2 & 2x\\ 0 & x^2\end{pmatrix}=\begin{pmatrix}x^3 & 3x^2\\ 0 & x^3\end{pmatrix}\begin{pmatrix}x^3 & 3x^{3-1}\\ 0 & x^3\end{pmatrix}$

$A^n=\begin{pmatrix}x^n & nx^{n-1}\\ 0 & x^n\end{pmatrix}$

Ans: (a)

70. The value of $\Delta=\begin{vmatrix}5^2 & 5^3 & 5^4\\ 5^3 & 5^4 & 5^5\\ 5^4 & 5^6 & 5^7\end{vmatrix}$ is

Options:

(a) $5^2$ (b) 0 (c) $5^{13}$ (d) $5^9$

Sol: $\Delta=5^2\cdot5^3\cdot5^4\begin{vmatrix}1 & 1 & 1\\ 5 & 5 & 5\\ 25 & 25 & 25\end{vmatrix}=0$

Ans: (b)

71. The maximum value of $\Delta=\begin{vmatrix}1 & 1 & 1\\ 1 & 1+\sin\theta & 1\\ 1+\cos\theta & 1 & 1\end{vmatrix}$ is

$(\theta \text{ is real numbers})$

Options:

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) $\frac{2\sqrt{3}}{4}$

Sol: We have, $\Delta=\begin{vmatrix}1 & 1 & 1\\ 1 & 1+\sin\theta & 1\\ 1+\cos\theta & 1 & 1\end{vmatrix}$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 0 & \sin\theta & 0 \\ \cos\theta & 0 & 0 \end{vmatrix} \begin{matrix} R_2 - R_1 \\ R_3 - R_1 \end{matrix}$$

$$\Delta = -\sin\theta\cos\theta = \frac{-1}{2}\sin 2\theta$$

$\sin 2\theta$ is max $2\theta = \frac{-\pi}{2} \Rightarrow \theta = \frac{-\pi}{4}$.

$\therefore$ Max value is $\frac{1}{2}$.

Ans: (a)

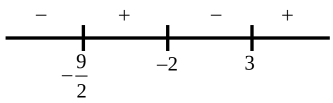
72. Solution set of the inequation $\frac{1}{x+2} < \frac{3}{x-3}$ is

Options:

(a) $\left(-\frac{9}{2}, 2\right) \cup (3, \infty)$ (b) $\left(-\infty, -\frac{9}{2}\right) \cup (2, 3)$ (c) $\left(-\frac{9}{2}, 2\right) \cup (2, 3)$ (d) $\left(-\infty, -\frac{9}{2}\right) \cup (3, \infty)$

Sol: $\frac{1}{x+2} < \frac{3}{x-3} \Rightarrow \frac{1}{x+2} - \frac{3}{x-3} < 0 \Rightarrow \frac{x-3-3x-6}{(x+2)(x-3)} < 0$

$\Rightarrow \frac{-(2x+9)}{(x+2)(x-3)} < 0 \Rightarrow \frac{2x+9}{(x+2)(x-3)} > 0$




$\therefore$ Solution set is $\left(-\frac{9}{2}, -2\right) \cup (3, \infty)$.

Ans: (a)

73. If $n$ is any positive integer then the value of $\frac{i^{4n+1} - i^{4n-1}}{2} =$

Options:

(a) 1 (b) $-1$ (c) $i$ (d) $-i$

Sol: $\frac{i^{4n+1} - i^{4n-1}}{2} = \frac{i - i^{-1}}{2}$ $\left(\because i^{4n} = 1\right)$

$= \frac{i - \frac{1}{i}}{2} = \frac{i+i}{2} = i$ $\left(\because \frac{1}{i} = -i\right)$

Ans: (c)

74. The equation of the line passing through $(1, 2)$ and perpendicular to $x + y + 7 = 0$ is

Options:

(a) $y - x + 1 = 0$ (b) $y - x - 1 = 0$ (c) $y - x + 2 = 0$ (d) $y - x - 2 = 0$

Sol: Slope of line $x + y + 7 = 0$ is $-1$

Slope of perpendicular line $m_p = \frac{-1}{m} = \frac{-1}{-1} = 1$

$\therefore$ Equation of perpendicular line is

$y - y_1 = m(x - x_1)$

$y - 2 = 1(x - 1)$

$x - y + 1 = 0$ i.e., $y - x - 1 = 0$

Ans: (b)

75. The major axis of an ellipse is three times the minor axis. Then the eccentricity is

Options:

(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{2}{3}$ (c) $\frac{\sqrt{2}}{3}$ (d) $\frac{1}{3}$

Sol: By data, $2a = 3(2b) \Rightarrow a = 3b$

Now, $e^2 = \frac{a^2 - b^2}{a^2} = \frac{9b^2 - b^2}{9b^2} = \frac{8}{9} \Rightarrow e = \frac{2\sqrt{2}}{3}$

Ans: (a)

76. If $\text{cosec } A + \cot A = \frac{11}{2}$, then $\tan A$ is

Options:

(a) $\frac{21}{22}$ (b) $\frac{15}{16}$ (c) $\frac{44}{117}$ (d) $\frac{117}{43}$

Sol: $\text{cosec } A + \cot A = \frac{1}{\text{cosec } A - \cot A} = \frac{11}{2}$

$\text{cosec } A + \cot A = \frac{11}{2}$ $\text{cosec } A - \cot A = \frac{2}{11}$

$\Rightarrow 2\cot A = \frac{11}{2} - \frac{2}{11} = \frac{117}{22} \Rightarrow \tan A = \frac{44}{117}$

Ans: (c)

77. The $A + B + C = 180°$ then $\sin 2A + \sin 2B + \sin 2C =$

Options:

(a) $4\sin A \cdot \sin B \cdot \sin C$ (b) $4\cos A \cdot \cos B \cdot \cos C$

(c) $2\sin A \cdot \sin B \cdot \sin C$ (d) $8\sin A \cdot \sin B \cdot \sin C$

Sol:

$GE = \sin 2A + \sin 2B + \sin 2C$

$= 2\sin(A+B)\cos(A-B) + 2\sin C\cos C$

$= 2\sin(\cos(A-B) + 2\sin C\cos C)$ $\sin(A+B) = \sin C$ $= 2\sin C[\cos(A-B) + \cos C]$

$= 2\sin C[\cos(A-B) - \cos(A+B)]$ $\cos C = -\cos(A+B)$

$= 2\sin C[-2\sin A\sin(-B)] = 4\sin A\sin B\sin C$

Ans: (a)

78. If $\cos\theta - 4\ \sin\theta = 1$, then $\sin\theta + 4\ \cos\theta =$

Options:

(a) $\pm 1$ (b) 0 (c) $\pm 2$ (d) $\pm 4$

Sol: If $a\cos\theta - b\ \sin\theta = c$, then $b\ \cos\theta + a\sin\theta = \sqrt{a^2 + b^2 - c^2}$

Here we have, $\cos\theta - 4\sin\theta = 1.$; $a = 1, b = 4, c = 1.$ $\therefore 4\cos\theta + \sin\theta = \sqrt{1+16-1} = \pm 4$

Or

$(4\cos\theta + \sin\theta)^2 =$

$= 16\cos^2\theta + \sin^2\theta + 8\ \sin\theta\cos\theta = 16\left(1-\sin^2\theta\right) + \left(1-\cos^2\theta\right) + 8\ \sin\theta\cos\theta$

$= 16 + 1 - \left[\cos^2\theta + 16\sin^2\theta - 8\sin\theta\cos\theta\right] = 17 - (\cos\theta - 4\sin\theta)^2 = 17 - 1 = 16 \quad \therefore (4\cos\theta + \sin\theta) = \pm 4$

Ans: (d)

79. If $\vec{a} = i - j + 2k, \vec{b} = 2i + 3j + k$ and $\vec{c} = i - k$ then the magnitude of $\vec{a} + 2\vec{b} - 3\vec{c}$ is

Options:

(a) $\sqrt{87}$ (b) $\sqrt{78}$ (c) $\sqrt{89}$ (d) $\sqrt{101}$

Sol: $\vec{a} = i - j + 2k, \vec{b} = 2i + 3j + k, \vec{c} = i - k$

$\vec{a} + 2\vec{b} - 3\vec{c} = (i + 4i - 3i) + (-j + 6j) + (2k + 2k + 3k) = 2i + 5j + 7k$ ; $\left|\vec{a} + 2\vec{b} - 3\vec{c}\right| = \sqrt{4 + 25 + 49} = \sqrt{78}$

Ans: (b)

80. If $|\vec{a}| = 4, |\vec{b}| = 2$ and angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $\left(\vec{a} \times \vec{b}\right)$ is

Options:

(a) 48 (b) 16 (c) $\vec{a}$ (d) 15

Sol: We have, $\left(\vec{a} \times \vec{b}\right)^2 = \left|\vec{a} \times \vec{b}\right|^2 = |\vec{a}|^2 \cdot |\vec{b}|^2 \cdot \sin^2\theta = 16 \cdot 4 \cdot \frac{1}{4} = 16$

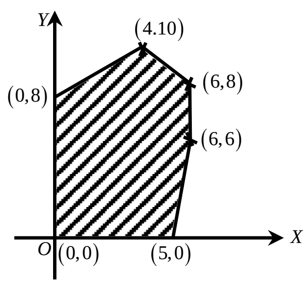
Ans: (b)

81. The feasible solution for a $LPP$ is shown in the following figure. Let $Z = 3x - 4y$, be the objective function. Maximum of $Z$ occurs at

Options:

(a) $(5,0)$ (b) $(6,5)$ (c) $(6,8)$ (d) $(4,10)$

Sol: Clearly, $Z_{(5,\,0)} = 3(5) - 4(0) = 15$ is maximum




Ans: (a)

82. The coordinates of the point $P=(3,4,5)$, then the direction cosines of $\vec{OP}$ are

Options:

(a) $3,4,5$ (b) $\frac{1}{3},\frac{1}{4},\frac{1}{5}$ (c) $\frac{3}{50},\frac{4}{50},\frac{1}{10}$ (d) $\frac{3}{5\sqrt{2}},\frac{4}{5\sqrt{2}},\frac{1}{\sqrt{2}}$

Sol: The direction ratios of $\vec{OP}$ are $3,4,5$.

$\therefore$ the direction cosines are $\frac{3}{\sqrt{9+16+25}},\frac{4}{\sqrt{9+16+25}},\frac{5}{\sqrt{9+16+25}}$

i.e., $\frac{3}{5\sqrt{2}},\frac{4}{5\sqrt{2}},\frac{1}{\sqrt{2}}$

Ans: (d)

83. The angle between the lines $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$ is

Options:

(a) $45°$ (b) $30°$ (c) $60°$ (d) $90°$

Sol: The lines are $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$

The d.r's of the lines are $2,5,4$ and $1,2,-3$

Clearly, $2\cdot(1)+5\cdot(2)+4\cdot(-3)=2+10-12=0$

Thus the angle between the lines if $90°$

Ans: (d)

84. $\lim_{x\to 0}\frac{1-\cos 5x}{\sin 4x}=$

Options:

(a) $\frac{5}{4}$ (b) $\frac{4}{5}$ (c) $0$ (d) $-\frac{5}{4}$

Sol: $\lim_{x\to 0}\frac{1-\cos 5x}{\sin 4x}=\lim_{x\to 0}\frac{5\sin 5x}{4\cos 4x}$ (LH Rule)

$=\frac{5}{4}\times\frac{0}{1}=0$

Ans: (c)

85. $\lim_{x\to 0}\frac{e^x-(1+x)}{x^2}=$

Options:

(a) $0$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $1$

Sol: $\lim_{x\to 0}\frac{e^x-(1+x)}{x^2}$ $\left(\frac{0}{0}\text{form}\right)$

$=\lim_{x\to 0}\frac{e^x-1}{2x}$ $\left(\frac{0}{0}\text{form}\right)$

$= \lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2}$

Ans: (c)

86. Let $f(x)=\begin{cases} \frac{3}{x^2}\sin 2x^2 & x<0 \\ \frac{x^2+2x+x}{1-3x^2} & x\ge 0, x\ne \frac{1}{\sqrt{3}}, \\ 0 & x=\frac{1}{\sqrt{3}} \end{cases}$ $f$ be continuous at $x=0$, then $c=$

Options:

(a) $-6$ (b) $6$ (c) $5$ (d) $-5$

Sol: $f$ is continuous at $x=0$

$\Rightarrow \lim_{x\to 0} f(x) = f(0) \Rightarrow \lim_{x\to 0} f(x) = c \quad (\therefore f(0)=c)$

Now. $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{3}{x^2}\cdot \sin 2x^2 = \lim_{x\to 0} 6\cdot\left(\frac{\sin 2x^2}{2x^2}\right) = 6$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} \frac{x^2+2x+c}{1-3x^2} = c$

$c=6$

Ans: (b)

87. Let $f(x)=|\cos x|$. Then

Options:

(a) $f$ is every where differentiable

(b) $f$ is every where continuous not differentiable at $x=n\pi, n\in Z$

(c) $f$ is every continuous but not differentiable at $x=(2n+1)\frac{\pi}{2}, n\in Z$

(d) None of these

Sol: Clearly, $f(x)=|\cos x|$ is continuous at every points but not differentiable at $x=(2n+1)\frac{\pi}{2}, n\in Z$.

Ans: (c)

88. $\frac{d}{dx}\left(\frac{3e^x+4}{2e^x-3}\right)=$

Options:

(a) $\frac{-17e^x}{(2e^x-3)^2}$ (b) $\frac{17e^x}{(2e^x-3)^2}$ (c) $\frac{e^x}{(2e^x-3)^2}$ (d) $\frac{e^x}{2e^x-3}$

Sol: $y=\frac{3e^x+4}{2e^x-3} \therefore \frac{(2e^x-3)(3e^x)-(3e^x+4)(2e^x)}{(2e^x-3)^2} = \frac{-17e^x}{(2e^x-3)^2}$

Ans: (a)

89. If $y=\sin^{-1}\left[\frac{1-x^2}{1+x^2}\right]$, then $\frac{dy}{dx}=$

Options:

(a) $-\frac{2}{1+x^2}$ (b) $\frac{2}{1+x^2}$ (c) $\frac{1}{2+x^2}$ (d) $\frac{2}{2-x^2}$

Sol: $y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$,

Put, $x=\tan\theta \Rightarrow \theta=\tan^{-1}x$

$\Rightarrow y=\sin^{-1}[\cos 2\theta)=\sin^{-1}\left[\sin\left(\frac{\pi}{2}-2\theta\right)\right] \Rightarrow y=\frac{\pi}{2}-2\tan^{-1}x \Rightarrow \frac{dy}{dx}=\frac{-2}{1+x^2}$

Ans: (a)

90. If $y=e^{\left(x^e\right)}$ then $\frac{dy}{dx}=$

Options:

(a) $e^{\left(x^2\right)}.\left(x^2\right)$ (b) $e^{\left(x^2\right)}\cdot x^2\log x$ (c) $e^{\left(x^e\right)}\cdot ex^{e-1}$ (d) None of these

Sol: $y=e^{\left(x^e\right)}$

$y=e^{\left(x^e\right)}$

$\therefore \frac{dy}{dx}=\frac{d}{dx}\left(e^{x^e}\right)=e^{x^e}\times\frac{d}{dx}\left(x^e\right)$

$=e^{x^e}\cdot e\cdot x^{e-1}$

Ans: (c)

91. If $y=(\sin x)^{\tan x}$, then $\frac{dy}{dx}=$

Options:

(a) $(\sin x)^{\tan x}\left[1+\sec^2 x\cdot\log\sin x\right]$

(b) $\tan x\cdot(\sin x)^{\tan x-1}$

(c) $\tan x\cdot(\sin x)^{\tan x-1}\cdot\cos x$

(d) $(\sin x)^{\tan x}\cdot\log(\sin x)\cdot\sec^2 x$

Sol: $y=(\sin x)^{\tan x} \Rightarrow \frac{dy}{dx}=(\sin x)^{\tan x}\left[\frac{\tan x}{\sin x}\cdot\cos x+\log(\sin x)\cdot\sec^2 x\right]$

$\Rightarrow \frac{dy}{dx}=(\sin x)^{\tan x}\left[1+\sec^2 x\cdot\log(\sin x)\right]$

Ans: (a)

92. A rod of length 13 meters has one end $P$ on the $x-$axis and the other end $Q$ on the $y-$axis. If $P$ moves along the $x-$axis with a speed of 12 m / sec, then the speed of the other end $Q$ when it is 12 meters from the origin is

Options:

(a) $-3$ m/sec (b) $-4$ m/sec (c) $-5$ m/sec (d) $-4$ m/sec

Sol: Let $OP = x, OQ = y \Rightarrow x^2 + y^2 = 169 \Rightarrow x = \sqrt{169 - y^2}$

$$\frac{dx}{dt} = \frac{-2y}{2\sqrt{169-y^2}}\frac{dy}{dt} \Rightarrow \frac{dy}{dt} = \frac{-\sqrt{169-y^2}}{y}\cdot(12) \quad \left(\therefore \frac{dx}{dt} = 12\right)$$

$$\Rightarrow \left(\frac{dy}{dt}\right)_{y=12} = \frac{-\sqrt{169-144}}{12}\times 12 = -5\text{ m/sec}$$

Ans: (c)

93. $\int_0^1 (x-1)e^{-x}\,dx =$

Options:

(a) 0 (b) $e$ (c) $\frac{1}{e}$ (d) $-\frac{1}{e}$

Sol: $I = \int_0^1 (x-1)e^{-x}\,dx = (x-1)\left(-e^{-x}\right) - 1\cdot e^{-x}\Big]_0^1$

$$-e^{-x}(x-1+1)\Big]_0^1 = -\left[xe^{-x}\right]_0^1 = -e^{-1} = -\frac{1}{e}$$

Ans: (d)

94. $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}dx = (a > 0)$

Options:

(a) 0 (b) $\pi$ (c) $\frac{\pi}{2}$ (d) $2\pi$

Sol: $I = \int_{-\pi}^{0} \frac{\cos^2 x}{1+a^x}\,dx + \int_0^{\pi} \frac{\cos^2 x}{1+a^x}dx$

Put $x = -t$ in the first integral

$\Rightarrow dx = -t$

$$I = -\int_{\pi}^{0} \frac{\cos^2 x}{1+a^{-x}}dx + \int_0^{\pi}\frac{\cos^2 x}{1+a^x}\,dx = \int_0^{\pi}\left(\frac{1}{1+a^{-x}} + \frac{1}{1+a^x}\right)\cos^2 x\,dx$$

$$= \int_0^{\pi}\left(\frac{a^x}{1+a^x} + \frac{1}{1+a^x}\right)\cos^2 x\,dx = \int_0^{\pi}\cos^2 x\,dx = 2\int_0^{\pi/2}\cos^2 x\,dx = 2.\frac{\pi}{4} = \frac{\pi}{2}$$

$\left(\therefore \cos^2 x \text{ is an even function}\right)$

Ans: (c)

95. $\int_1^2 \frac{dx}{x\left(1+x^4\right)} =$

Options:

(a) $\frac{1}{4}\log\left(\frac{17}{32}\right)$ (b) $\frac{1}{4}\log\left(\frac{17}{2}\right)$ (c) $\log\left(\frac{17}{2}\right)$ (d) $\frac{1}{4}\log\left(\frac{32}{17}\right)$

Sol: $\int \frac{dx}{x\left(1+x^n\right)} = \frac{1}{n}\log\left(\frac{x^n}{1+x^n}\right)$; $\int_1^2 \frac{dx}{x\left(1+x^4\right)} = \frac{1}{4}\log\left(\frac{x^4}{1+x^4}\right)\Bigg]_1^2 = \frac{1}{4}\left[\log\frac{16}{17} - \log\frac{1}{2}\right] = \frac{1}{4}\log\left(\frac{32}{17}\right)$

Ans: (d)

96. The value of $\int_{-\pi/2}^{\pi/2}\left(x^3 + x\cos x + \tan^5 x + 1\right) dx$

Options:

(a) 0 (b) 2 (c) $\pi$ (d) 1

Sol: We have $I = \int_{-\pi/2}^{\pi/2}\left(x^3 + x\cos x + \tan^5 x\right)dx + \int_{-\pi/2}^{\pi/2} 1dx = 0 + [x]_{-\pi/2}^{\pi/2} = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$(\because f(x)\text{ is odd function})$

Ans: (c)

97. $\int \frac{\sin^6 x}{\cos^8 x}\, dx =$

Options:

(a) $-\frac{\tan^7 x}{7} + C$ (b) $\frac{\tan^7 x}{7} + C$ (c) $\frac{7}{\cos^7 x} + C$ (d) $\frac{1}{7\cos^7 x} + C$

Sol: $\int \frac{\sin^6 x}{\cos^8 x}\, dx = \int \frac{\sin^6 x}{\cos^6 x}\cdot\frac{1}{\cos^2 x}\, dx = \int \tan^6 x \cdot \sec^2 x\, dx = \frac{\tan^7 x}{7} + C$

Ans: (b)

98. $\int e^x\left(\frac{1+\sin x\cdot\cos x}{1+\cos 2x}\right) dx =$

Options:

(a) $e^x \tan x$ (b) $\frac{1}{2}e^x \tan x$ (c) $\frac{1}{2}e^x \cot x$ (d) $2e^x \tan x$

Sol: $I = \int e^x\left(\frac{1+\sin x\ \cos x}{2\cos^2 x}\right)dx = \frac{1}{2}\int e^x\left(\sec^2 x + \tan x\right)dx = \frac{1}{2}\int e^x\left(\tan x + \sec^2\right)dx = \frac{1}{2}e^x \tan x + c$

Ans: (b)

99. $\int \frac{dx}{(x+3)(x-3)} =$

Options:

(a) $\frac{1}{3}\log\left(\frac{x+3}{x-3}\right) + C$ (b) $\frac{1}{6}\log(3x) + C$ (c) $\frac{1}{6}\log\left(\frac{x-3}{x}\right) + C$ (d) $\frac{1}{6}\log\left(\frac{x-3}{x+3}\right) + C$

Sol: $\int \frac{dx}{(x+3)(x-3)} = \int \frac{dx}{x^2-9} = \int \frac{dx}{x^2-3^2} = \frac{1}{2(3)}\log\left(\frac{x-3}{x+3}\right)+c = \frac{1}{6}\log\left(\frac{x-3}{x+3}\right)+c\left[\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)\right]$

Ans: (d)

100. The differential equation for $y = A\cos\alpha x + B\sin\alpha x$ where $A$ and $B$ are arbitrary constants is

Options:

(a) $\frac{d^2y}{dx^2} - \alpha^2 y = 0$ (b) $\frac{d^2y}{dx^2} + \alpha^2 y = 0$ (c) $\frac{d^2y}{dx^2} + \alpha y = 0$ (d) $\frac{d^2y}{dx^2} - \alpha y = 0$

Sol: We have, $y = A\cos\alpha x + B\sin\alpha x$

$\Rightarrow \frac{dy}{dx} = \alpha(-A\sin\ \alpha x + B\ \cos\ \alpha x)$

$\Rightarrow \frac{d^2y}{dx^2} = \alpha^2(-A\cos\alpha x - B\sin\alpha x) = -\alpha^2 y \Rightarrow \frac{d^2y}{dx^2} + \alpha^2 y = 0$

Ans: (b)

101. The general solution of $\frac{dy}{dx} = 2xe^{x^2-y}$ is

Options:

(a) $e^{x^{2-y}} = c$ (b) $e^{-y} + e^{x^2} = c$ (c) $e^y = e^{x^2} + c$ (d) $e^{x^2+y} = c$

Sol: We have, $\frac{dy}{dx} = 2x \cdot e^{x^2} \cdot e^{-y}$

$\Rightarrow \int e^y dy = \int e^{x^2} \cdot 2x\ dx + c \Rightarrow e^y = e^{x^2} + c$

Ans: (c)

102. A die is thrown and a card is selected at random from a deck of $52$ playing cards. The probability of getting an even number on the die and a spade card is

Options:

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{167}{168}$

Sol: Required probability $=$ $P(\text{an even number}) \times P(\text{a spade}) \frac{3}{6} \times \frac{13}{52} = \frac{1}{8}$

($\because$ only even numbers are $2, 4, 6$ and there are $13$ spades out of $52$ cards).

Ans: (c)

103. In a college of $30$ students fail in physics, $25$ fail in mathematics and $10$ fail in both. One student is chosen at random. The probability that she fails in physics, if she failed in mathematics is

Options:

(a) $\frac{1}{10}$ (b) $\frac{2}{5}$ (c) $\frac{9}{20}$ (d) $\frac{1}{3}$

Sol: Set, $E:$ event student fails in physics

$F:$ event student fails in Mathematics

Now, $P(E) = \frac{30}{100} = \frac{3}{10}$ and $P(F) = \frac{25}{100} = \frac{1}{4}$

$P(E\cap F)=\frac{10}{100}=\frac{1}{10}$

Required probability $=P(E/F)=\frac{P(E\cap F)}{P(F)}=\frac{(1/10)}{1/4}=\frac{2}{5}$

Ans: (b)

104. $A$ and $B$ are two students. Their chances of solving a problem correctly are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If the probability of their making a common error is $\frac{1}{20}$ and they obtain the same answer, then the probability of their answer to be correct is

Options:

(a) $\frac{1}{12}$ (b) $\frac{1}{40}$ (c) $\frac{13}{120}$ (d) $\frac{10}{13}$

Sol: Let $E$: event $A$ and $B$ obtain same answer

$F$: event Both $A$ and $B$ obtain correct answer

$\Rightarrow F\subset E$ ;

$P(E)=P(F)+P$ (both $A$ and $B$ make same error and get same incorrect answer)

$=\left(\frac{1}{3}\times\frac{1}{4}\right)+\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\times\frac{1}{20}=\frac{1}{12}+\frac{1}{40}=\frac{13}{120}$

$P(F)=\frac{1}{3}\times\frac{1}{4}=\frac{1}{12}$

Required probability $=P(F/E)=\frac{P(E\cap F)}{P(E)}=\frac{P(F)}{P(E)}$ $(\because F\subset E)$

$=\frac{(1/12)}{(13/120)}=\frac{10}{13}$

Ans: (d)

105. Which of the following is correct?

(a) $A\cap\phi=A$ (b) $A\cap\phi=\phi$ (c) $A\cap\phi=U$ (d) $A\cap\phi=A'$

Sol: $A\cap\phi=\phi$ is correct

Ans: (b)

106. If $P=\begin{bmatrix}1&\alpha&3\\1&3&3\\2&4&4\end{bmatrix}$ is the adjoint of a $3\times 3$ matrix $A$ and $|A|=4$, then $\alpha$ is equal to

(a) 4 (b) 11 (c) 5 (d) 0

Sol: $|P|=1(12-12)-\alpha(4-6)+3(4-6)=2\alpha-6$

$\because\quad P=adj(A)$

$\therefore\quad |P|=|adj\ A|=|A|^2=16\Rightarrow\ 2\alpha-6=16\ \Rightarrow\ 2\alpha=22\Rightarrow\alpha=11$

Ans: (b)

107.If the sum of series $\sum_{n=0}^{\infty} r^n = S$ for $|r|<1$, then the sum of the series $\sum_{n=0}^{\infty} r^n$ , is

(a) $S^2$ (b) $\frac{S^2}{2S+1}$ (c) $\frac{2S}{S^2-1}$ (d) $\frac{S^2}{2S-1}$

Sol: Since, $1+r+r^2+....\infty = S$

$\therefore \quad \frac{1}{1-r} = S \Rightarrow r = \frac{S-1}{S} \quad \ldots$ (i)

Now, $1+r+r^2+....\infty = \frac{1}{1-r^2} = \frac{1}{1-\left(\frac{S-1}{S}\right)^2}$ [From Eq.(i)]

$= \frac{S^2}{S^2-(S-1)^2} = \frac{S^2}{(2S-1)}$

Ans: (d)

108.The sum of the coefficients in the expansion of $\left(1+x-3x^2\right)^{3148}$ is

(a) 8 (b) 7 (c) 1 (d) $-1$

Sol: On substituting $x=1$ in $\left(1+x-3x^2\right)^{3148}$, we get

$=(1+1-3)^{3148} = (-1)^{3148} = 1$.

Ans: (c)

109.If the system of equations $x+ky-z=0$, $3x-ky-z=0$ and $x-3y+z=0$, has non-zero solution, then $k$ is equal to

(a) $-1$ (b) 0 (c) 1 (d) 2

Sol: The system has non-zero solution if

$$\begin{vmatrix} 1 & k & -1 \\ 3 & -k & -1 \\ 1 & -3 & 1 \end{vmatrix} = 0$$

$\Rightarrow 1(-k-3)-k(3+1)-1(-9+k)=0 \Rightarrow -6k+6=0$

$\therefore \quad k=1$

Ans: (c)

110.The function $f(x) = x - \cot x$

(a) always increases (b) always decreases

(c) never decreases (d) sometimes increases and sometimes decreases

Sol: We have, $f(x) = x - cotx$

$f'(x) = 1+\cos ec^2 x$ Evidently, $1+\cos ec^2 x > 0$

Thus, the function $f(x) = x - \cot x$ always increases

Ans: (a)

111.The value of $\cos 15^\circ \cos 7\frac{1^\circ}{2} \sin 7\frac{1^\circ}{2}$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{16}$

Sol: We have, $\cos 15^\circ \cos 7\frac{1^\circ}{2} \sin 7\frac{1^\circ}{2} = \frac{1}{2}\cos 15^\circ \sin 15^\circ$

$= \frac{1}{4}\sin 30^\circ = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

Ans: (b)

112.The value of $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$ is

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

Sol: $\sin 50^\circ + \sin 10^\circ - \sin 70^\circ$

$= 2\sin 30^\circ \cos 20^\circ - \cos 20^\circ = \cos 20^\circ \left(2 \times \frac{1}{2} - 1\right) = 0$

Ans: (a)

113.The projection of $a = 3\hat{i} - \hat{j} + 5\hat{k}$ on $b = 2\hat{i} + 3\hat{j} + \hat{k}$ is

(a) $\frac{8}{\sqrt{35}}$ (b) $\frac{8}{\sqrt{39}}$ (c) $\frac{8}{\sqrt{14}}$ (d) $\sqrt{14}$

Sol: The projection of $a$ on $b = \frac{a \cdot b}{|b|}$

$= \frac{(3\hat{i} - \hat{j} + 5\hat{k}) \cdot (2\hat{i} + 3\hat{j} + \hat{k})}{\sqrt{2^2 + 3^2 + 1^2}} = \frac{8}{\sqrt{14}}$

Ans: (c)

114.If the direction cosines of two lines are such that $l + m + n = 0, l^2 + m^2 - n^2 = 0$, then the angle between them is

(a) $\pi$ (b) $\pi/3$ (c) $\pi/4$ (d) $\pi/6$

Sol: Given, $l + m + n = 0$ … (i)

And $l^2 + m^2 - n^2 = 0$ … (ii)

$\therefore \quad l^2 + m^2 - (-l - m)^2 = 0$

$\Rightarrow \; l^2 + m^2 - l^2 - m^2 + 2lm = 0 \quad \Rightarrow \quad 2lm = 0 \Rightarrow l = 0 \;\; or \;\; m = 0$

If $l = 0$, then $n = -m$

$\Rightarrow \quad l : m : n = 0 : 1 : -1$

And if $m = 0$, then $n = -l$

$\Rightarrow \quad l : m : n = 1 : 0 := -1$

$\therefore \; \cos\theta = \frac{0 + 0 + 1}{\sqrt{0+1+1}\sqrt{0+1+1}} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

Ans: (b)

115.The difference between two numbers is 48 and the difference between their arithmetic mean and their geometric mean is 18. Then the greater of two numbers is

(a) 96 (b) 60 (c) 54 (d) 49

Sol: Let the two numbers be $a$ and $b$, such that $a > b$.

$\therefore \quad a-b=48$ and $\frac{a+b}{2}-\sqrt{ab}=18$

$\Rightarrow \quad \left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)=48$

And $\left(\sqrt{a}-\sqrt{b}\right)=6$

$\Rightarrow \quad \sqrt{a}+\sqrt{b}=8$ and $\left(\sqrt{a}-\sqrt{b}\right)=6$

$\Rightarrow \quad \sqrt{a}=7$ and $\sqrt{b}=1 \quad \Rightarrow \quad a=49$ and $b=1$

Ans: (d)

116.On the interval $[0,1]$, the function $x^{25}(1-x)^{75}$ takes its maximum value at the point

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

Sol: Given, $f(x)=x^{25}(1-x)^{75}$

$\Rightarrow \quad f'(x)=25x^{24}(1-x)^{75}-75x^{25}(1-x)^{74}$

$=25^{24}(1-x)^{74}(1-4x)$

Put $f'(x)=0, \quad x=0,1 \; and \; 1/4$

If $x<\frac{1}{4}$, then $f'(x)=25x^{24}(1-x)^{74}(1-4x)>0$

And $if \quad x>\frac{1}{4}$, then $f'(x)=25x^{24}(1-x)^{74}(1-4x)<0$

Thus, $f'(x)$ changes its sign from positive to negative as $x$ passes through $1/4$ from left to right.

Hence, $f(x)$ attains its maximum at $x=1/4$.

Ans: (b)

117.If the radius of a circle is increasing at a uniform rate of $2\,cm/s$. The area of increasing of area of circle, at the instant when the radius is $20cm$, is

(a) $70\pi\,cm^2/s$ (b) $70\,cm^2/s$ (c) $80\pi\,cm^2/s$ (d) $80\,cm^2/s$

Sol: Let area of circle, $A=\pi r^2$

$\therefore \quad \frac{dA}{dt}=2\pi r\frac{dr}{dt}=2\pi\cdot 20\cdot 2=80\pi\,cm^2/s$

Ans: (c)

118.If $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$, then the value of $x$ is

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) None of these

Sol: $\therefore\ \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right)=\tan^{-1}(2\cos ecx)$

$$\Rightarrow \frac{2\cos x}{1-\cos^2 x}=2\cos ecx \Rightarrow \frac{2\cos x}{\sin^2 x}=2\cos ecx \qquad \Rightarrow \sin x=\cos x \Rightarrow x=\frac{\pi}{4}$$

Ans: (b)

119.The number of real solutions of $\tan^{-1}\left\{\sqrt{x(x+1)}\right\}+\sin^{-1}\left\{\sqrt{x^2+x+1}\right\}=\frac{\pi}{2}$, is

(a) 0 (b) 1 (c) 2 (d) $\infty$

Sol: Given, $\tan^{-1}\left\{\sqrt{x(x+1)}\right\}=\frac{\pi}{2}-\sin^{-1}\left\{\sqrt{x^2+x+1}\right\}$

$$\Rightarrow \cos^{-1}\left\{\frac{1}{\sqrt{1+\left[\sqrt{x(x+1)}\right]^2}}\right\}=\cos^{-1}\left\{\sqrt{x^2+x+1}\right\}$$

$$\Rightarrow \frac{1}{\sqrt{1+x(x+1)}}=\sqrt{x^2+x+1}\Rightarrow 1=x^2+x+1$$

$$\Rightarrow x^2+x=0\Rightarrow x(x+1)=0\Rightarrow x=0\ or\ x=-1$$

Ans: (c)

120.The value of $\cos^{-1}\left(-\frac{1}{2}\right)$ among the following, is

(a) $\frac{9\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{5\pi}{3}$ (d) $\frac{11\pi}{3}$

Sol: Let $\theta=\cos^{-1}\left(-\frac{1}{2}\right)$

$$\Rightarrow \cos\theta=-\frac{1}{2}=-\cos\left(\frac{\pi}{3}\right)=\cos\left(\pi-\frac{\pi}{3}\right)=\cos\left(\frac{2\pi}{3}\right)$$

$$\Rightarrow \theta=\frac{2\pi}{3}$$

Ans: (b)

## Physics

**Multiple Choice Questions with one correct answer. A correct answer carries 1 mark. No negative mark. 60 x 1 = 60**

121. The maximum and minimum distances of a comet from the sun are $8\times10^{12}$ m and $1.6\times10^{12}$ m respectively. If its velocity when nearest to the sun is $60\,\text{ms}^{-1}$, what will be its velocity in $\text{ms}^{-1}$ when it is farthest?

(a) 12 (b) 60 (c) 112 (d) 6

Sol: By law of conservation of angular momentum, $mvr = \text{constant}$

$$v_{\min}\times r_{\max} = v_{\max}\times r_{\min} \quad \therefore\ v_{\min} = \frac{60\times1.6\times10^{12}}{8\times10^{12}} = 12\,\text{ms}^{-1}$$

Ans: (a)

122. A steel cable with a radius 2 cm supports a chairlift at a ski area. If the maximum stress is not to exceed $10^8\ \text{Nm}^{-2}$, the maximum load the cable can support is

(a) $4\pi\times10^5$ N (b) $4\pi\times10^4$ N (c) $2\pi\times10^5$ N (d) $2\pi\times10^4$ N

Sol: Here, $r = 2\,\text{cm} = 2\times10^{-2}$ m

Maximum load = maximum stress $\times$ Area of cross-section

$$=10^8\ \text{Nm}^{-2}\times\pi\times\left(2\times10^{-2}\text{m}\right)^2 = 4\pi\times10^4\ \text{N}$$

Ans: (b)

123. A ring of radius 0.5 m and mass 10 kg is rotating about its diameter with angular velocity of $20\ \text{rad s}^{-1}$. Its rotational kinetic energy is

(a) 10 J (b) 100 J (c) 500 J (d) 250 J

Sol: Rotational kinetic energy $=\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\times\omega^2$

$$=\frac{1}{2}\left(\frac{1}{2}\times10\times(0.5)^2\right)\times(20)^2 = 250\ \text{J}$$

Ans: (d)

124. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator the length of water column in the capillary tube will be

(a) 10 cm (b) 8 cm (c) 20 cm (d) 4 cm

Sol: Water fills the tube entirely in gravity less condition i.e., 20 cm

Ans: (c)

125. When the temperature of a rod increases from $t$ to $(t+\Delta t)$, its moment of inertia increases from $I$ to $(I+\Delta I)$. If $\alpha$ be the coefficient of llinear expansion of the rod, then the value of $\frac{\Delta I}{I}$ is

(a) $2\alpha\Delta t$ (b) $\alpha\Delta t$ (c) $\frac{\alpha\Delta t}{2}$ (d) $\frac{\Delta t}{2}$

Sol: Moment of inertia of a rod,

$$I = \frac{1}{12}ML^2 \qquad \ldots \text{(i)}$$

where $M$ is the mass of the rod and $L$ is the length of the rod

$$\therefore \ \Delta I = \frac{1}{2}2ML\Delta L \qquad [\because M \text{ is a constant}] \qquad \ldots \text{(ii)}$$

Divide (ii) by (i), we get

$$\frac{\Delta I}{I} = 2\frac{\Delta L}{L} \qquad \ldots \text{(iii)}$$

$\because \ \Delta L = L\alpha\Delta t$

or $\frac{\Delta L}{L} = \alpha\Delta t$

Substituting the value of $\frac{\Delta L}{L}$ in (iii), we get $\frac{\Delta I}{I} = 2\alpha\Delta t$

Ans: (a)

126.The pressure is $P$, volume $V$ and temperature $T$ of a gas in jar $A$ and the other gas in jar $B$ is at pressure $P$, volume $V/4$ and temperature $2T$, then the ratio of the number of molecules in jar $A$ and $B$ will be

(a) $1:1$ (b) $1:2$ (c) $2:1$ (d) $8:1$

Sol: $PV = NkT \ \Rightarrow \ \frac{N_A}{N_B} = \frac{P_A V_A}{V_B V_B} \times \frac{T_B}{T_A}$

$$\Rightarrow \frac{N_A}{N_B} = \frac{P \times V \times (2T)}{P \times \frac{V}{4} \times T} = \frac{8}{1}$$

Ans: (d)

127.Two moles of helium gas $(\gamma = 5/3)$ are initially at temperature $27\,°\text{C}$ and occupy a volume of $20$ litres. The gas is first expanded at constant pressure until the volume is doubled. Then, it undergoes an adiabatic change until the temperature returns to the initial value. What is the final volume of the gas?

(a) $113.13$ lit (b) $115.2$ lit (c) $120$ lit (d) $125$ lit

Sol: For a perfect gas, $PV = \mu RT$

$$P_1 = \frac{\mu RT}{V} = \frac{2 \times 8.31 \times (273 + 27)}{20 \times 10^{-3}}$$

$$P_1 = 2.5 \times 10^5 \ \text{Nm}^{-2}$$

At constant pressure, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$$\therefore \ T_2 = \left(\frac{V_2}{V_1}\right)T_1 = 2 \times 300 = 600\,\text{K}$$

The gas now undergoes an adiabatic change.

$T_1 = 600\,\text{K}, \ T_2 = 300\,\text{K}, \ V_1 = 40\,\text{lit.}, V_2 = ?$

$\gamma - 1 = 5/3 - 1 = 2/3$

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$600(40)^{2/3} = 300(V_2)^{2/3}$

$(2)^{3/2} \times 40 = V_2$ or $V_2 = 113.13$ lit.

Ans: (a)

128. Two equations of two S.H.M. are $x = a\sin(\omega t - \alpha)$ and $y = b\cos(\omega t - \alpha)$. The phase difference between the two is

(a) $0°$ (b) $\alpha°$ (c) $90°$ (d) $180°$

Sol: $y = a\sin(\omega t - \alpha) = a\cos\left(\omega t - \alpha - \frac{\pi}{2}\right)$

Another equation is given $y = \cos(\omega t - \alpha)$

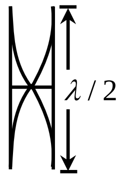
So, there exists a phase difference of $\frac{\pi}{2} = 90°$

Ans: (c)

129. The ratio of fundamental frequency of an organ pipe opened at both ends to that of the organ pipe closed at one end is

(a) $1:1$ (b) $1.5:1$ (c) $2:1$ (d) $3:1$

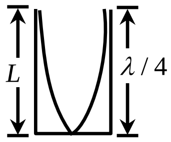
Sol: The fundamental frequency of an organ pipe open at both ends is




$n_0 = \frac{v}{2L}$ ... (i)

The fundamental frequency of an organ pipe closed at one end is

$n_c = \frac{v}{4L}$ ... (ii)




Dividing equation (i) by (ii)

$\frac{n_0}{n_c} = \frac{v}{2L} \times \frac{4L}{v} = \frac{2}{1}$

Ans: (c)

130. The charges on two spheres are $+7\mu C$ and $5\mu C$ respectively. They experience a force $F$. If each of them is given and additional charge of $-2\mu C$, the new forces of attraction will be

(a) $F$ (b) $F/2$ (c) $F/\sqrt{3}$ (d) $2F$

Sol: $F = \frac{1}{4\pi\varepsilon_0}\frac{(+7\times10^{-6})(-5\times10^{-6})}{r^2} = -\frac{1}{4\pi\varepsilon_0}\frac{35\times10^{12}}{r^2}$ N

$F' = \frac{1}{4\pi\varepsilon_0}\frac{(+5\times10^{-6})(-7\times10^{-6})}{r^2} = -\frac{1}{4\pi\varepsilon_0}\frac{35\times10^{12}}{r^2}$ N

Ans: (a)

131. A charge $Q$ is enclosed by a Gaussian spherical surface of radius $R$. If the radius is doubled, then the outward electric flux will

(a) increase four times
(b) be reduced to half
(c) remain the same
(d) be doubled

Sol: By Gauss's theorem, $\phi = \frac{Q_{in}}{\varepsilon_0}$

Thus, the net flux depends only on the charge enclosed by the surface. Hence, there will be no effect on the net flux if the radius of the surface is doubled.

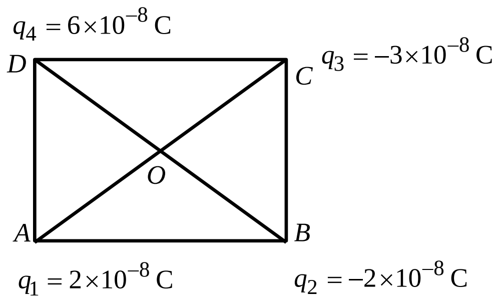
Ans: (c)

132. Four charges $q_1 = 2\times10^{-8}$C, $q_2 = -2\times10^{-8}$C, $q_3 = -3\times10^{-8}$C, and $q_4 = 6\times10^{-8}$C are placed at four corners of a square of side $\sqrt{2}$ m. What is the potential at the centre of the square?

(a) 270 V
(b) 300 V
(c) zero
(d) 100 V

Sol: $AC = BD = \sqrt{\left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2} = 2\,\text{m}$

$\therefore\ DO = OB = AO = OC = \frac{2}{2} = 1\,\text{m}$




$\therefore$ Potential at the centre $O$, $V = k\frac{q}{r}$

$$V = k\left[\frac{2\times10^{-8}}{1} + \frac{-2\times10^{8}}{1} + \frac{-3\times10^{-8}}{1} + \frac{6\times10^{-8}}{1}\right]$$

$V = k\times3\times10^{-8} = 9\times10^{9}\times3\times10^{-8}$ volt

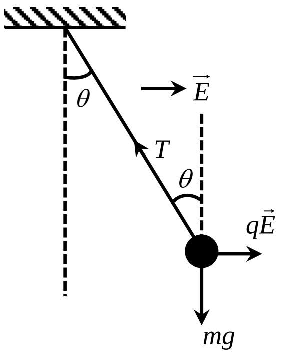
$V = 27\times10 = 270$ volt

Ans: (a)

133. A pendulum bob of mass $30.7\times10^{-6}$ kg carrying a charge $2\times10^{-8}$ C is at rest in a horizontal uniform electric field of $20000\ \text{Vm}^{-1}$. The tension in the thread of the pendulum is $\left(g = 9.8\,\text{ms}^{-2}\right)$

(a) $3\times10^{-4}$ N
(b) $4\times10^{-4}$ N
(c) $5\times10^{-4}$ N
(d) $6\times10^{-4}$ N

Sol: At equilibrium




$T\cos\theta = mg$ and $T\sin\theta = qE$

$mg = 30.7\times10^{-6}\times9.8 = 3\times10^{-4}$ N

$qE = 2\times10^{-8}\times20000 = 4\times10^{-4}$ N

$\therefore\ T = \sqrt{\left(3\times10^{-4}\right)^2 + \left(4\times10^{-4}\right)^2}$

$= 5\times10^{-4}$ N

Ans: (c)

134.The electric potential at a point $(x, y, z)$ is given by $V = -x^2y - xz^3 + 4$. The electric field $\vec{E}$ at that point is

(a) $\vec{E} = \hat{i}2xy + \hat{j}\left(x^2 + y^2\right) + \hat{k}\left(3xz - y^2\right)$ (b) $\vec{E} = \hat{i}z^3 + \hat{j}xyz + \hat{k}z^2$

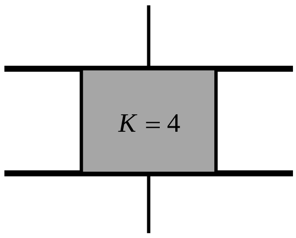
(c) $\vec{E} = \hat{i}\left(2xy - z^3\right) + \hat{j}xy^2 + \hat{k}3z^2x$ (d) $\vec{E} = \hat{i}\left(2xy + z^3\right) + \hat{j}x^2 + \hat{k}3xz^2$

Sol: The electric field at a point is equal to negative potential gradient at that point.

$$\vec{E} = -\frac{\partial V}{\partial r} = \left[-\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}\right] = \left[\left(2xy + z^3\right)\hat{i} + \hat{j}x^2 + \hat{k}\,3x\,z^2\right]$$

Ans: (d)

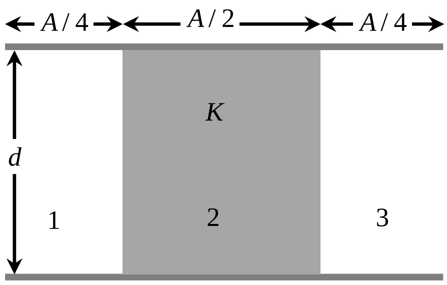
135.Consider a parallel plate capacitor of $10\,\mu\text{F}$ (micro-farad) with air filled in the gap between the plates. Now one half of the space between the plates is filled with a dielectric of dielectric constant $4$, as shown in the figure. The capacity of the capacitor changes to




(a) 25 μF (b) 20 μF (c) 40 μF (d) 5 μF

Sol:

$$C_1 = \frac{\varepsilon_0\left(\frac{A}{4}\right)}{d},\quad C_2 = \frac{K\varepsilon_0\left(\frac{A}{2}\right)}{d},\quad C_3 = \frac{\varepsilon_0\left(\frac{A}{4}\right)}{d}$$




$$C_{eq} = C_1 + C_2 + C_3 = \left(\frac{K+1}{2}\right)\frac{\varepsilon_0 A}{d} = \left(\frac{4+1}{2}\right)\times 10$$

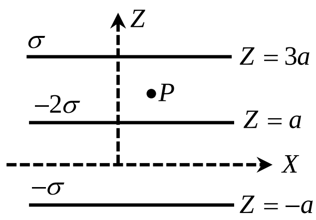
$= 25\ \mu\text{F}$

Ans: (a)

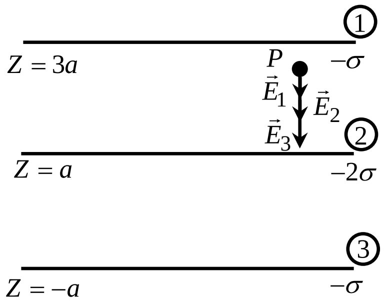
136.Three infinitely long charge sheets are placed as shown in figure. The electric field at point $P$ is

(a) $\frac{2\sigma}{\varepsilon_0}\hat{k}$ (b) $\frac{4\sigma}{\varepsilon_0}\hat{k}$

(c) $-\frac{2\sigma}{\varepsilon_0}\hat{k}$ (d) $-\frac{4\sigma}{\varepsilon_0}\hat{k}$



Sol: Figure shows the electric fields due to the sheets $1, 2$ and $3$ at point $P$. The direction of electric fields are according to the charge on the sheets (away from positively charge sheet and towards the negatively charged sheet and perpendicular).




The total electric field, $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

$$= E_1(-\hat{k}) + E_2(-\hat{k}) + E_3(-\hat{k}) = \left[\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right](-\hat{k}) = -\frac{2\sigma}{\varepsilon_0}\hat{k}$$

Ans: (c)

137. The electric field intensity just sufficient to balance the earth's gravitational attraction on an electron will be: (given mass and charge of an electron respectively are $9.1\times10^{-31}$ kg, $1.6\times10^{-19}$ C and $g = 10\,\text{m s}^{-2}$)

(a) $-5.6\times10^{-11}\ \text{NC}^{-1}$ (b) $-4.8\times10^{-15}\ \text{NC}^{-1}$

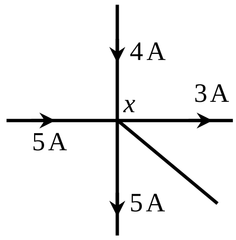
(c) $-1.6\times10^{-19}\ \text{NC}^{-1}$ (d) $-3.2\times10^{-19}\ \text{NC}^{-1}$

Sol: $-eE = mg$

$$\vec{E} = -\frac{9.1\times10^{-31}\times10}{1.6\times10^{-19}} = -5.6\times10^{11}\ \text{NC}^{-1}$$

Ans: (a)

138. Five conductors are meeting at a point $x$ as shown in the figure. What is the value of current in fifth conductor



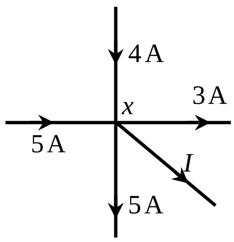

(a) 3 A away from $x$

(b) 1A away from $x$

(c) 4 A away from $x$

(d) 1 A away from $x$

Sol: According to Kirchhoff's first law,




$(+5\,\text{A}) + (+4\,\text{A}) + (-3\,\text{A}) + (-5\,\text{A}) + I$

$\Rightarrow\ I = -1\,\text{A}$

$-$ve sign shows that current is flowing away from $x$

Ans: (b)

139. An electric current passes through a circuit containing two wires of the same material connected in parallel. If the lengths of the wires are in the ratio of $4/3$ and radius of the wires are in the ratio of $2/3$, then the ratio of the currents passing through the wires will be

(a) 3 (b) $1/3$ (c) $3/9$ (d) None of these

Sol: Given: $\frac{l_1}{l_2} = \frac{4}{3}$ and $\frac{r_1}{r_2} = \frac{2}{3}$

Since the two wires are connected in parallel, potential remains same. i.e.,

$V = \text{constant}$

$IR = \text{Constant}$

i.e., $I_1R_1 = I_2R_2 \Rightarrow \frac{I_1}{I_2} = \frac{R_2}{R_1}$ ... (i)

But we know that, $R = \frac{\rho l}{A}$ $\therefore \frac{R_1}{R_2} = \left(\frac{l_1}{A_1}\right)\left(\frac{A_2}{l_2}\right) = \left(\frac{l_1}{l_2}\right)\left(\frac{A_1}{A_2}\right)$

$= \left(\frac{l_1}{l_2}\right)\left(\frac{r_1}{r_2}\right)^2$ (since area, $A = \pi r^2$)

$= \left(\frac{4}{3}\right)\left(\frac{3}{2}\right)^2 = 3$

Substitute this value in equation (i)

we get, $\frac{I_1}{I_2} = \frac{1}{3}$

Ans: (b)

140. When the current $i$ is flowing through a conductor, the drift velocity is $v$. If $2i$ current flows through the same metal but having double the area of cross-section, then the drift velocity will be

(a) $\frac{v}{4}$ (b) $\frac{v}{2}$ (c) $v$ (d) $4v$

Sol: $v_d = \frac{J}{ne} \Rightarrow v_d \propto J$ [current density]

$J_1 = \frac{i}{A}$ and $J_2 = \frac{2i}{2A} = \frac{1}{A}J_1$;

$\therefore (v_d)_1 = (v_d)_2 = v$

Ans: (c)

141. A small power station supplies electricity to 5000 lamps connected in parallel. Each lamp has a resistance of 220 ohm and is operated at 220 V. The total current supplied by the station is

(a) 2500 A (b) 3500 A (c) 5000 A (d) 10000 A

Sol: All the lamps have been connected in parallel. Therefore, each operates at the same voltage of 220 V.

$\therefore$ Current drawn by each lamp

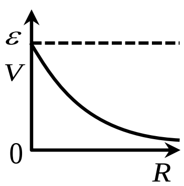
$= \frac{220}{220} = 1\text{A}$

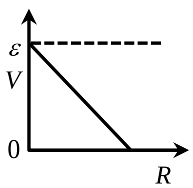
$\therefore$ Total current drawn by lamps

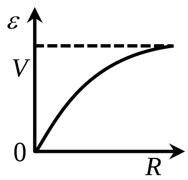
$= 5000\text{ A}$

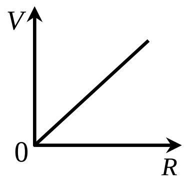
Ans: (c)

142. Cell having an emf $\varepsilon$ and internal resistance $r$ is connected across a variable external resistance $R$. As the resistance $R$ is increased, the plot of potential difference $V$ across $R$ is given by

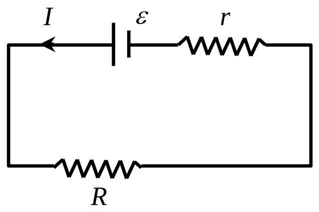
(a) 


(b) 


(c) 


(d) 

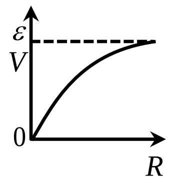
Sol: The current through the resistance $R$




$$I = \left(\frac{\varepsilon}{R+r}\right)$$

The potential difference across $R$

$$V = IR = \left(\frac{\varepsilon}{R+r}\right)R$$




$$V = \frac{\varepsilon}{\left(1+\frac{r}{R}\right)}$$

when $R = 0,\ V = 0,$

$R = \infty,\ V = \varepsilon$

Thus $V$ increases as $R$ increases upto certain limit, but it does not increase further.

Ans: (c)

143. The resistance of a bulb filmnet is $100\,\Omega$ at a temperature of $100\,°\text{C}$. If its temperature of coefficient be $0.005$ per $°\text{C}$, its resistance will become $200\,\Omega$ at a temperature of

(a) 300°C (b) 400°C (c) 500°C (d) 200°C

Sol: $R_1 = R_0\left[1+\alpha\times 100\right] = 100$ … (i)

$R_2 = R_0\left[1+\alpha\times T\right] = 200$ … (ii)

On dividing we get

$$\frac{200}{100} = \frac{1+\alpha T}{1+100\alpha} \Rightarrow 2 = \frac{1+0.005T}{1+100\times 0.005}$$

$\Rightarrow T = 400\,°\text{C}$

Ans: (b)

144. An electron enters a region where magnetic field $(B)$ and electric field $(E)$ are mutually perpendicular, then

(a) it will always move in the direction of $B$ (b) it will always move in the direction of $E$

(c) it always possesses circular motion (d) it can go undeflected also

Sol:

When electrons enter in a region where there is only magnetic field, then force exerted by the magnetic field will deflect the electron in a direction perpendicular to its motion and also perpendicular to the magnetic field.

Whereas in the presence of electric field, force exerted by the electric field is in a direction opposite to the direction of electric field. As a result, electron deflected by magnetic field is nullified by the deflection by electric field. As a result, electron moves un-deflected.

Ans: (d)

145. Magnetic field intensity at the centre of a coil of $50$ turns, radius $0.5\,\text{m}$ and carrying a current of $2\,\text{A}$ is

(a) $0.5\times 10^{-5}\,\text{T}$ (b) $1.25\times 10^{-4}\,\text{T}$ (c) $3\times 10^{-5}\,\text{T}$ (d) $4\times 10^{-5}\,\text{T}$

Sol: We know that magnetic field at the centre of circular coil,

$$B = \frac{\mu_0 I n}{2r} = \frac{4\pi \times 10^{-7} \times 2 \times 50}{2 \times 0.5}$$

$$= 1.25 \times 10^{-4}\ \text{T}$$

Ans: (b)

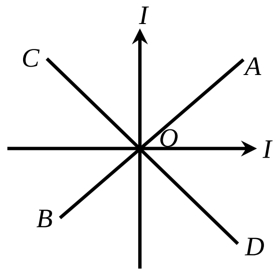
146. A straight wire of length 0.5 metre and carrying a current of 1.2 ampere is placed in uniform magnetic field of induction 2 tesla. The magnetic field is perpendicular to the length of the wire. The force on the wire is

(a) 2.4 N (b) 1.2 N (c) 3.0 N (d) 2.0 N

Sol: $F = Bi\ell = 2 \times 1.2 \times 0.5 = 1.2\ \text{N}$

Ans: (b)

147. Two equal electric currents are flowing perpendicular to each other as shown in the figure. $AB$ and $CD$ are perpendicular to each other and symmetrically placed with respect to the current flow. Where do we expect the resultant magnetic field to be zero?




(a) on $AB$

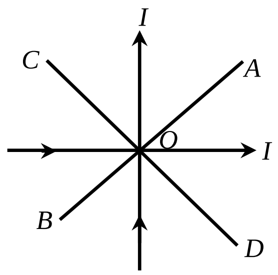
(b) on $CD$

(c) on both $AB$ and $CD$

(d) on both $OD$ and $BO$

Sol:

Net magnetic field on $AB$ is zero because magnetic field due to both current carrying wires is equal in magnitude but opposite in direction.




Ans: (a)

148. The magnetic lines of force inside a bar magnet

(a) are from $N$ – pole to $S$ – pole of magnet

(b) do not exist

(c) depend upon the area of cross section of bar magnet

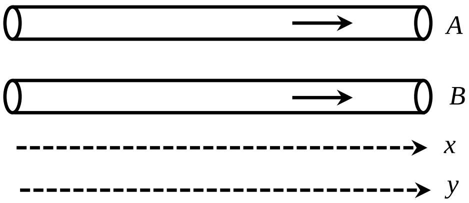
(d) are from $S$ – pole of magnet

Sol:

are from $S$ – pole of magnet

Ans: (d)

149. $A$ and $B$ are two conductors carrying a current $i$ in the same direction. $x$ and $y$ are two electron beams moving in the same direction. Then




(a) there will be repulsion between $A$ and B, attraction between $x$ and $y$

(b) there will be attraction between $A$ and B, repulsion between $x$ and $y$

(c) there will be repulsion between $A$ and B and also $x$ and $y$

(d) there will be attraction between $A$ and B and also $x$ and $y$

Sol: Current carrying conductors will attract each other, while electron beams will repel each other.

Ans: (b)

150. If a diamagnetic substance is brought near north or south pole of a bar magnet, it is

(a) attracted by the poles

(b) repelled by the poles

(c) repelled by north pole and attracted by the south pole

(d) attracted by the north pole and repelled by the south pole

Sol: Diamagnetic substances do not have any unpaired electron. And they magnetised in direction opposite to that of magnetic field. Hence, when they are brought to north or south pole of a bar magnet, they are repelled by poles.

Ans: (b)

151. A square coil of side $25\text{ cm}$ having 1000 turns is rotated with a uniform speed in a magnetic field about an axis perpendicular to the direction of the field. At an instant $t$, the emf induced in the coil is $e = 200\sin 100\pi t$ . The magnetic field is

(a) $0.50\,\text{T}$ (b) $0.02\,\text{T}$ (c) $0.01\,\text{T}$ (d) $0.1\,\text{T}$

Sol: $e = 200\sin 100\pi t$

$\therefore\ e_0 = 200, \omega = 100\pi$

Now, $NAB\omega = e_0$

$$\therefore\ B = \frac{e_0}{NA\omega} \text{ or } B = \frac{200}{1000\left(25\times10^{-2}\right)^2 \times 100\pi} \text{ or } B = 0.01\text{T}$$

Ans: (c)

152. The magnetic potential energy stored in a certain inductor is $25\,\text{mJ}$ , when the current in the inductor is $60\,\text{mA}$ . This inductor is of inductance

(a) $0.138\,\text{H}$ (b) $138.88\,\text{H}$ (c) $13.89\,\text{H}$ (d) $1.389\,\text{H}$

Sol: From question energy stored in inductor,

$U = 25\times10^{-3}\,\text{J}$

Current, $I = 60\text{ mA}$

Energy stored in inductor

$U = \frac{1}{2}LI^2$

$25\times10^{-3} = \frac{1}{2}\times L\times\left(60\times10^{-3}\right)^2$

$L = \frac{25\times2\times10^6\times10^{-3}}{3600} = 13.89\text{ H}$

Ans: (c)

153. A resistance of $20\,\text{ohm}$ is connected to a source of an alternating potential $V = 200\cos(100\pi t)$. The time taken by the current to change from its peak value to rms value, is

(a) $2.5\times10^{-3}\,\text{s}$ (b) $25\times10^{-3}\,\text{s}$ (c) $0.25\,\text{s}$ (d) $0.20\,\text{s}$

Sol: The current and potential difference are in phase with the resistance. So, the time taken would be same as time for voltage to change from $(t=0)$ that is peak value to rms value.

Time taken by voltage to achieve its rms value of $\frac{200}{\sqrt{2}}$

$\frac{200}{\sqrt{2}} = 200\cos(100\pi t) \Rightarrow \cos(100\pi t) = \frac{1}{\sqrt{2}} = \cos\left(\frac{\pi}{4}\right)$

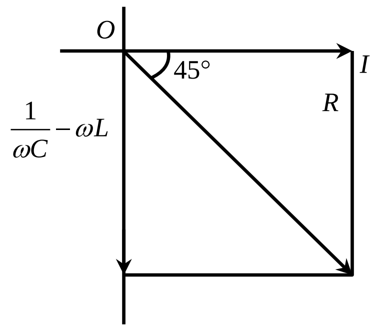
$t = \frac{1}{400}\text{second} = 2.5\times10^{-3}\,\text{s}$

Ans: (a)

154. In a circuit, $L, C$ and $R$ are connected in series with an alternating voltage source of frequency $f$. The current leads the voltage by $45^\circ$. The value of $C$ is

(a) $\frac{1}{\pi f(2\pi fL - R)}$ (b) $\frac{1}{2\pi f(2\pi fL - R)}$ (c) $\frac{1}{\pi f(2\pi fL + R)}$ (d) $\frac{1}{2\pi f(2\pi fL + R)}$

Sol: From figure,




$\tan 45^\circ = \frac{\frac{1}{\omega C} - \omega L}{R}$

$\Rightarrow \frac{1}{\omega C} - \omega L = R$

$\Rightarrow \frac{1}{\omega C} = R + \omega L$

$C = \frac{1}{\omega(R+\omega L)}$

$= \frac{1}{2\pi f(R + 2\pi fL)}$

Ans: (d)

155. A transformer is used to light a $100\,\text{W}$ and $110\,\text{V}$ lamp from a $220\,\text{V}$ mains. If the main current is $0.5\,\text{A}$, the efficiency of the transformer is approximately

(a) 50% (b) 90% (c) 10% (d) 30%

Sol: Efficiency of the transformer

$$\eta = \frac{P_{\text{output}}}{P_{\text{input}}} \times 100$$

$$= \frac{100}{220 \times 0.5} \times 100 = 90.9\%$$

Ans: (b)

156. The electric and the magnetic field associated with an E.M. wave, propagating along the $+z-$axis, can be represented by

(a) $\left[\vec{E} = E_0\hat{i},\ \vec{B} = B_0\hat{j}\right]$ (b) $\left[\vec{E} = E_0\hat{k},\ \vec{B} = B_0\hat{i}\right]$

(c) $\left[\vec{E} = E_0\hat{j},\ \vec{B} = B_0\hat{i}\right]$ (d) $\left[\vec{E} = E_0\hat{j},\ \vec{B} = B_0\hat{k}\right]$

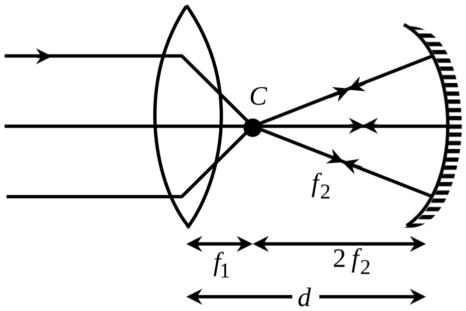
Sol: E.M. wave always propagates in a direction perpendicular to both electric and magnetic fields. So, electric and magnetic fields should be along $+X-$ and $+Y-$directions respectively. Therefore, option (a) is the correct option.

Ans: (a)

157. A concave mirror of focal length $'f_1'$ is placed at a distance of $'d'$ from a convex lens of focal length $'f_2'$. A beam of light coming from infinity and falling on this convex-lens concave mirror combination returns to infinity. The distance $'d'$ must be equal to

(a) $f_1 + f_2$ (b) $-f_1 + f_2$ (c) $2f_2 + f_1$ (d) $-2f_1 + f_2$

Sol: $d = 2f_2 + f_1$




Ans: (c)

158. A ray of light is incident at an angle of incidence, $i$, on one face of prism of angle $A$ (assumed to be small) and emerges normally from the opposite face. If the refractive index of the prism is $\mu$, the angle of incidence $i$, is nearly equal to

(a) $\mu A$ (b) $\frac{\mu A}{2}$ (c) $\frac{A}{\mu}$ (d) $\frac{A}{2\mu}$

Sol: For normally emerge $e = 0$

Therefore $r_2 = 0$ and $r_1 = A$

Snell's law for incident ray's

$1\sin i = \mu \sin r_1 = \mu \sin A$

For small angle

$i = \mu A$

Ans: (a)

159.When a biconvex lens of glass having refractive index 1.47 is dipped in a liquid, it acts as a plane sheet of glass. This implies that the liquid must have refractive index

(a) equal to that of glass
(b) less than one
(c) greater than that of glass
(d) less than that of glass

Sol: $\frac{1}{f} = \left(\frac{\mu_g}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

If $\mu_g = \mu_m$, then $\frac{1}{f} = (1-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\Rightarrow \frac{1}{f} = 0$

$f = \frac{1}{0} = \infty$

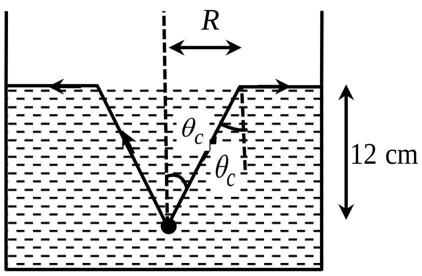
This implies that the liquid must have refractive index equal to glass.

Ans: (a)

160.A fish looking up through the water sees the outside world contained in a circular horizon. If the refractive index of water is $\frac{4}{3}$ and the fish is 12 cm below the surface, the radius of this circle in cm is

(a) $\frac{36}{\sqrt{7}}$
(b) $36\sqrt{7}$
(c) $4\sqrt{5}$
(d) $36\sqrt{5}$

Sol: $\sin\theta_c = \frac{1}{\mu} = \frac{3}{4}$




or $\tan\theta_c = \frac{3}{\sqrt{16-9}}$

$= \frac{3}{\sqrt{7}} = \frac{R}{12}$

$\Rightarrow R = \frac{36}{\sqrt{7}}$ cm

Ans: (a)

161.Two identical light waves, propagating in the same direction, have a phase difference $\delta$. After they superimpose, the intensity of the resulting wave will be proportional to

(a) $\cos\delta$
(b) $\cos(\delta/2)$
(c) $\cos^2(\delta/2)$
(d) $\cos^2\delta$

Sol: Here $A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$

$\because a_1 = a_2 = a$

$\because\ A^2 = 2a^2(1+\cos\delta) = 2a^2\left(1+2\cos^2\frac{\delta}{1}-1\right)$

$\Rightarrow\ A^2 \propto \cos^2\frac{\delta}{2}$

Now, $I \propto A^2$

$\therefore\ I \propto A^2 \propto \cos^2\frac{\delta}{2}$

$\therefore\ I \propto \cos^2\frac{\delta}{2}$

Ans: (c)

162. The locus of all particles in a medium, vibrating in the same phase is called

(a) Wavelet (b) fringe (c) wave front (d) None of these

Sol: Light is an electromagnetic wave of wavelength range $4000\,\text{Å}$ to $7800\,\text{Å}$

Ans: (c)

163. A steel ball of mass $m$ is moving with a kinetic energy $K$. The de-Broglie wavelength associated with the ball is

(a) $\frac{h}{2mK}$ (b) $\sqrt{\frac{h}{2mK}}$ (c) $\frac{h}{\sqrt{2mK}}$ (d) None of these

Sol: de-Broglie's relation, $\lambda = \frac{h}{p}$

momentum $p = \sqrt{2mE}$

$\Rightarrow\ \lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mK}} \qquad (\because\ E = K)$

Ans: (c)

164. All electrons ejected from a surface by incident light of wavelength $200\,\text{nm}$ can be stopped before travelling $1\,\text{m}$ in the direction of uniform electric field of $4\ \text{NC}^{-1}$. The work function of the surface is

(a) $4$ eV (b) $6.2$ eV (c) $2$ eV (d) $2.2$ eV

Sol: The electron ejected with maximum speed $v_{\max}$ are stopped by electric field $E = 4\ \text{NC}^{-1}$

After travelling a distance $d = 1\,\text{m}$

$\frac{1}{2}mv_{\max}^2 = eED = 4\,\text{eV}$

The energy of incident photon $= \frac{1240}{200} = 6.2\,\text{eV}$

From equation of photo electric effect

$\frac{1}{2}mv_{\max}^2 = hv - \phi$

$\therefore\ \phi_0 = 6.2 - 4 = 2.2\ \text{eV}$

Ans: (d)

165.In Rutherford's $\alpha-$particle scattering experiment, what will be correct angle for $\alpha$ scattering for an impact parameter $b=0$?

(a) 90° (b) 270° (c) 0° (d) 180°

Sol: When $b=0$, scattering angle $\theta=180°$

Ans: (d)

166.According to the Bohr theory of $H-$atom, the speed of the electron, its energy and the radius of its orbit varies with the principal quantum number $n$, respectively, as

(a) $\frac{1}{n}, n^2, \frac{1}{n^2}$ (b) $n, \frac{1}{n^2}, n^2$ (c) $n, \frac{1}{n^2}, \frac{1}{n^2}$ (d) $\frac{1}{n}, \frac{1}{n^2}, n^2$

Sol: According to Bohr's theory of hydrogen atom,

(i) The speed of the electron in the $n^{th}$ orbit is $v_n = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_0(h/2\pi)}$ or $v_n \propto \frac{1}{n}$

(ii) The energy of the electron in the $n^{th}$ orbit is $E_n = -\frac{me^4}{8n^2\varepsilon_0^2 h^2} = -\frac{13.6}{n^2}\text{eV}$ or $E_n \propto \frac{1}{n^2}$

(iii) The radius of the electron in the $n^{th}$ orbit is $r_n = \frac{n^2h^2\varepsilon_0}{\pi mc^2} = n^2 a_0$ where $a_0 = \frac{h^2\varepsilon_0}{\pi me} = 5.29\times10^{11}\text{ m}$ is called Bohr's radius, or $r_n \propto n^2$.

Ans: (d)

167.Energy of an electron in an excited hydrogen atom is $-3.4\,\text{eV}$. Its angular momentum will be

(a) $3.72\times10^{-34}$ Js (b) $2.10\times10^{-34}$ Js (c) $1.51\times10^{-34}$ Js (d) $4.20\times10^{-34}$ Js

Sol:

$$E_n = -\frac{13.6\text{ eV}}{n^2}$$

$$\Rightarrow 3.4 = -\frac{13.6}{n^2}$$

$$\Rightarrow n^2 = 4 \text{ or } n = 2$$

Now, $L_n = \frac{nh}{2\pi}$

$$\therefore L = \frac{2h}{2\pi} = \frac{h}{\pi} = \frac{6.62}{3.14} = 2.10\times10^{-34}\text{ Js}$$

$$= 2.10\times10^{-34}\text{ Js}$$

Ans: (b)

168. $M_n$ and $M_p$ represent mass of neutron and proton respectively. If an element having atomic mass $M$ and $N-$neutrons and $Z-$protons, then the correct relation will be

(a) $M < [NM_n + ZM_p]$ (b) $M > [NM_n + ZM_p]$

(c) $M = [NM_n + ZM_p]$ (d) $M = N[M_n + M_p]$

Sol: Actual mass of the nucleus is always less than total mass of nucleons, so $M < \left(NM_n + Zm_p\right)$

Ans: (a)

169.The binding energy per nucleon for ${}_1^2H$ and ${}_2^4He$ respectively are $1.1\text{ MeV}$ and $7.1\text{ MeV}$. The energy released in MeV when two ${}_1^2H$ nuclei to form ${}_2^4He$ is

(a) 4.4 (b) 8.2 (c) 24 (d) 28.4

Sol: The chemical reaction of process is $2{}_1^2H \rightarrow {}_2^4He$

Energy released $= 4\times(7.1) - 4(1.1) = 24\text{ MeV}$

Ans: (c)

170.Nuclear force exists between

(a) Neutron-neutron (b) Proton-proton (c) Neutron-proton (d) all of these

Sol: All of these

Ans: (d)

171.When germanium is doped 1 part in a million with indium, its conductivity increases by a factor of about

(a) 10 (b) $10^3$ (c) $10^5$ (d) $10^6$

Sol: The introduction of indium in the proportion of 1 in $10^6$ (million) increases the conductivity by a factor of $10^6$.

Ans: (d)

172.Pure $Si$ at $500\text{ K}$ has equal number of electron $(n_e)$ and hole $(n_h)$ concentrations of $1.5\times10^{16}\text{ m}^{-3}$. Doping by indium increases $n_h$ to $4.5\times10^{22}\text{ m}^{-3}$. The doped semiconductor is of

(a) $n-$type with electron concentration $n_e = 5\times10^{22}\text{ m}^{-3}$

(b) $p-$type with electron concentration $n_e = 2.5\times10^{10}\text{ m}^{-3}$

(c) $n-$type with electron concentration $n_e = 2.5\times10^{23}\text{ m}^{-3}$

(d) $p-$type having electron concentration $n_e = 5\times10^{9}\text{ m}^{-3}$

Sol: $n_i^2 = n_e n_h$

$\left(1.5\times10^{16}\right)^2 = n_e\left(4.5\times10^{22}\right)$

$\Rightarrow n_e = 0.5\times10^{10}$ or $\Rightarrow n_e = 5\times10^{9}$

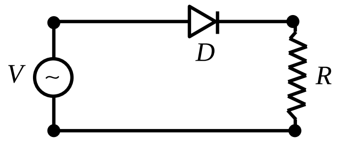
Given, $n_h = 4.5\times10^{22}$ $\Rightarrow n_h >> n_e$

$\therefore$ Semiconductor is $p-$type and

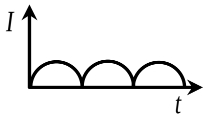
$n_e = 5\times10^{9}\text{ m}^{-3}$

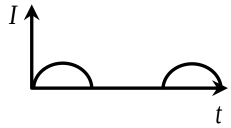
Ans: (d)

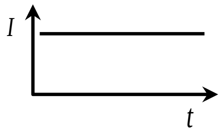
173.A $p-n$ junction $(D)$ shown in the figure can act as a rectifier. An alternative current source $(V)$ is connected in the circuit.

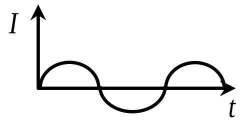



The current $(I)$ in the resistor $(R)$ can be shown by

(a) 


(b) 


(c) 


(d) 


Sol:

We know that a single $p-n$ junction diode connected to an $a-c$ source acts as a half wave rectifier. [Forward biased in one half cycle and reverse biased in the other half cycle].

Ans: (b)

174.A charged particle with charge $q$ enters a region of constant, uniform and mutually orthogonal fields $\vec{E}$ and $\vec{B}$ with a velocity $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$, and comes out without any change in magnitude or direction of $\vec{v}$. Then

(a) $\vec{v} = \vec{B}\times\vec{E}/E^2$ (b) $\vec{v} = \vec{E}\times\vec{B}/B^2$ (c) $\vec{v} = \vec{B}\times\vec{E}/B^2$ (d) $\vec{v} = \vec{E}\times\vec{B}/E^2$

Sol: Here, $\vec{E}$ and $\vec{B}$ are perpendicular to each other and the velocity $\vec{v}$ does not change; therefore

$qE = qvB \Rightarrow v = \dfrac{E}{B}$

Also, $\left|\dfrac{\vec{E}\times\vec{B}}{B^2}\right| = \dfrac{EB\sin\theta}{B^2} = \dfrac{EB\sin 90°}{B^2} = \dfrac{E}{B} = |\vec{v}| = v$

Ans: (b)

175.If momentum $(P)$, area $(A)$ and time $(T)$ are taken to be fundamental quantities, then the energy has the dimensional formula

(a) $\left[P^1A^{-1}T^1\right]$ (b) $\left[P^2A^1T^1\right]$ (c) $\left[P^1A^{-1/2}T^1\right]$ (d) $\left[P^1A^{1/2}T^{-1}\right]$

Sol: Let energy $E = kp^aA^bt^c$ … (i)

Where $k$ is a dimensionless constant of proportionality.

Equating dimensions on both sides of (i), we get

$$\left[ML^2T^{-2}\right] = \left[MLT^{-1}\right]^a\left[M^0L^2T^0\right]^b\left[M^0L^0T\right]^c = \left[M^aL^{a+2b}T^{-a+c}\right]$$

Applying the principle of homogeneity of dimensions, we get

$a = 1$ … (ii)

$a+2b=2$ ... (iii)

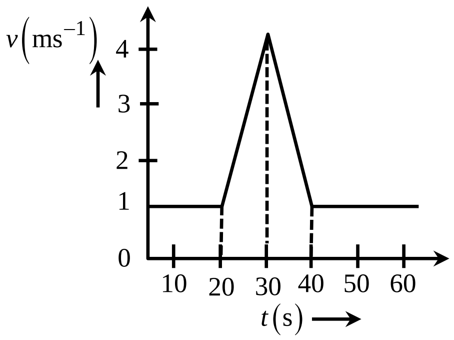
$-a+c=-2$ ... (iv)

On solving eqs. (ii), (iii) and (iv), we get $a=1, b=\frac{1}{2}, c=-1$

$\therefore [E]=\left[p^1 A^{1/2} t^{-1}\right]$

Ans: (d)

176. Velocity time $(v-t)$ graph for a moving object is shown in the figure. Total displacement of the object during the time interval when there is non-zero acceleration and retardation is




(a) 60 m (b) 50 m (c) 30 m (d) 40 m

Sol:

Between time interval 20 s to 40 s, there is non-zero acceleration and retardation. Hence distance travelled during this interval = Area between time interval 20 s to 40 s

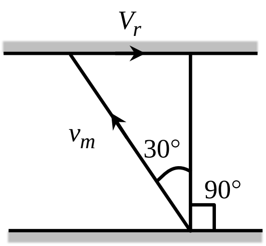
$=\frac{1}{2}\times 20\times 3+20\times 1=30+20=50\,\text{m}$

Ans: (b)

177. A person aiming to reach the exactly opposite point on the bank of a stream is swimming with a speed of $0.5\,\text{ms}^{-1}$ at an angle of $120°$ with the direction of flow of water. The speed of water in the stream is

(a) $1\text{ ms}^{-1}$ (b) $0.5\,\text{ms}^{-1}$ (c) $0.25\,\text{ms}^{-1}$ (d) $0.433\,\text{ms}^{-1}$

Sol: $\sin 30° = \frac{v_r}{v_m} = \frac{1}{2}$




$\Rightarrow v_r = \frac{v_m}{2} = \frac{0.5}{2} = 0.25\text{ ms}^{-1}$

Ans: (c)

178. A conveyor belt is moving at a constant speed of $2\ \text{ms}^{-1}$. A box is gently dropped on it. The coefficient of friction between them is $\mu = 0.5$. The distance that the box will move relative to belt before coming to rest on it taking $g = 10\,\text{ms}^{-2}$, is

(a) 1.2 m (b) 0.6 m (c) zero (d) 0.4 m

Sol: Frictional force on the box $f = \mu mg$

$\therefore$ Acceleration in the box

$a = \mu g = 5\,\text{ms}^{-2}$

$v^2 = u^2 + 2as$

$\Rightarrow 0 = 2^2 + 2\times(5)s$

$\Rightarrow s = -\frac{2}{5}$ w.r.t. belt

$\Rightarrow$ distance $= 0.4\,\text{m}$

Ans: (d)

179. A body of mass $5\,\text{kg}$ is moving with a momentum of $10\,\text{kg ms}^{-1}$. A force of $0.2\,\text{N}$ acts on it in the direction of motion of the body for $10$ second. The increase in its kinetic energy is

(a) $4.4\,\text{J}$ (b) $3.8\,\text{J}$ (c) $3.2\,\text{J}$ (d) $2.8\,\text{J}$

Sol: Initial momentum, $p_1 = 10\ \text{kg ms}^{-1}$

Change in momentum for force $0.2\,\text{N}$ for $10$ second,

$\Delta p = ft = (0.2\times10)\,\text{kg ms}^{-1} = 2\,\text{kg ms}^{-1}$

Change in momentum,

$p_2 = (10+2) = 12\,\text{kg ms}^{-1}$

Initial velocity, $v_1 = \frac{p_1}{m} = \frac{10}{5} = 2\ \text{ms}^{-1}$

$v_2 = \frac{p_2}{m} = \frac{12}{5}\text{ms}^{-1}$

Change in kinetic energy

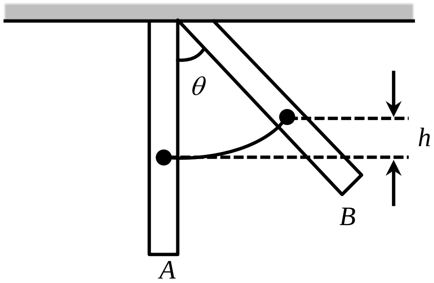
$\Delta k = \frac{1}{2}m\left(v_2^2 - v_1^2\right) = \frac{1}{2}\times5\times\left[\left(\frac{12}{5}\right) - (2)^2\right]$

$= \frac{1}{2}\times5\left(\frac{144}{25} - 4\right) = 4.4\ \text{J}$

Ans: (a)

180. A thin uniform rod of length $l$ and mass $m$ is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is $\omega$. Its centre of mass rises to a maximum height of

(a) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (b) $\frac{1}{6}\frac{l\omega}{g}$ (c) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (d) $\frac{1}{6}\frac{l^2\omega^2}{g}$




Sol: Rotational KE at $A$ position $= \frac{1}{2} I\omega^2$

Potential energy at $B$ position $= mgh$

By the law of conservation of energy

$$mgh = \frac{1}{2} I\omega^2 \Rightarrow mgh = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$$

$$h = \frac{l^2\omega^2}{6g}$$

Ans: (d)

**Key Answers**:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. c | 2. d | 3. d | 4. b | 5. a | 6. d | 7. d | 8. b | 9. a | 10. c |
| 11. b | 12. a | 13. b | 14. c | 15. d | 16. d | 17. a | 18. b | 19. d | 20. d |
| 21. d | 22. c | 23. a | 24. a | 25. c | 26. c | 27. d | 28. b | 29. a | 30. d |
| 31. a | 32. a | 33. b | 34. b | 35. a | 36. a | 37. b | 38. b | 39. d | 40. b |
| 41. d | 42. b | 43. b | 44. d | 45. a | 46. c | 47. a | 48. d | 49. c | 50. d |
| 51. b | 52. a | 53. b | 54. b | 55. c | 56. c | 57. c | 58. c | 59. d | 60. a |
| 61. c | 62. b | 63. c | 64. b | 65. a | 66. b | 67. d | 68. c | 69. a | 70. b |
| 71. a | 72. a | 73. c | 74. b | 75. a | 76. c | 77. a | 78. d | 79. b | 80. b |
| 81. a | 82. d | 83. d | 84. c | 85. c | 86. b | 87. c | 88. a | 89. a | 90. c |
| 91. a | 92. c | 93. d | 94. c | 95. d | 96. c | 97. b | 98. b | 99. d | 100.b |
| 101.c | 102.c | 103.b | 104.d | 105.b | 106.b | 107.d | 108.c | 109.c | 110.a |
| 111.b | 112.a | 113.c | 114.b | 115.d | 116.b | 117.c | 118.b | 119.c | 120.b |
| 121.a | 122.b | 123.d | 124.c | 125.a | 126.d | 127.a | 128.c | 129.c | 130.a |
| 131.c | 132.a | 133.c | 134.d | 135.a | 136.c | 137.a | 138.b | 139.b | 140.c |
| 141.c | 142.c | 143.b | 144.d | 145.b | 146.b | 147.a | 148.d | 149.b | 150.b |
| 151.c | 152.c | 153.a | 154.d | 155.b | 156.a | 157.c | 158.a | 159.a | 160.a |
| 161.c | 162.c | 163.c | 164.d | 165.d | 166.d | 167.b | 168.a | 169.c | 170.d |
| 171.d | 172.d | 173.b | 174.b | 175.d | 176.b | 177.c | 178.d | 179.a | 180.d |