

1. If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx}$ is

(a) 2^{y-x}

(b) -2^{y-x}

(c) 2^{x-y}

(d) $\frac{2^y - 1}{2^x - 1}$

$$\text{Sol: } \frac{dy}{dx} = \frac{2^x(1-2^y)}{2^y(2^x-1)}$$

$$= \frac{2^x - (2^x - 2^y)}{2^x + 2^y - 2^y}$$

$$= -2^{y-x}$$

Ans: (b)

2. If $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $f'(\sqrt{3})$ is

(a) $-\frac{1}{2}$

(b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{3}}$

(d) $-\frac{1}{\sqrt{3}}$

$$\text{Sol: } f(x) = 2 \tan^{-1} x$$

$$f'(x) = \frac{2}{1+x^2}$$

$$f'(\sqrt{3}) = \frac{2}{4} = \frac{1}{2}$$

Ans: (b)

3. The right hand and left hand limit of the function $f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ are respectively

(a) 1 and 1

(b) 1 and -1

(c) -1 and -1

(d) -1 and 1

$$\text{Sol: } \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} \frac{e^{1/h}-1}{e^{1/h}+1}$$

$$= \lim_{h \rightarrow 0} \frac{1-e^{-1/h}}{1+e^{-1/h}} = 1$$

$$\lim_{h \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} \frac{e^{-1/h}-1}{e^{-1/h}+1} = \frac{0-1}{0+1} = -1$$

Ans: (b)

4. If $y = 2x^{n+1} + \frac{3}{x^n}$, then $x^2 \frac{d^2y}{dx^2}$ is

(a) $6n(n+1)y$

(b) $n(n+1)y$

(c) $x \frac{dy}{dx} + y$

(d) y

$$\text{Sol: } \frac{dy}{dx} = 2(n+1)x^n - \frac{3n}{x^{n+1}}$$

$$\frac{d^2y}{dx^2} = 2n(n+1)x^{n-1} + \frac{3n(n+1)}{x^{n+2}}$$

$$\frac{x^2 d^2 y}{dx^2} = n(n+1) \left[2x^{n+1} + \frac{3}{x^n} \right] = n(n+1)y$$

Ans: (b)

5. If the curves $2x = y^2$ and $2xy = K$ intersect perpendicularly, then the value of K^2 is

(a) 4

(b) $2\sqrt{2}$

(c) 2

(d) 8

$$\text{Sol: } 2x = y^2, 2xy = k$$

$$\frac{dy}{dx} = \frac{1}{y}, \frac{dy}{dx} = \frac{-y}{x}$$

$$m_1 m_2 = -1$$

$$\frac{1}{y} \times \frac{y}{x} = -1$$

$$X = 1$$

$$\therefore y^2 = 2$$

$$4x^2 y^2 = k^2$$

$$k^2 = 4(1)(2)$$

$$k^2 = 8$$

Ans: (d)

6. If $(xe)^y = e^y$, then $\frac{dy}{dx}$ is

$$(a) \frac{\log x}{(1+\log x)^2}$$

$$(b) \frac{1}{(1+\log x)^2}$$

$$(c) \frac{\log x}{(1+\log x)}$$

$$(d) \frac{e^x}{x(y-1)}$$

$$\text{Sol: } (xe)^y = e^x$$

Take log on both sides

$$y(1+\log x) = x$$

$$y = \frac{x}{1+\log x}$$

$$\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$$

Ans: (a)

7. If the side of a cube is increased by 5%, then the surface area of a cube is increased by

(a) 10%

(b) 60%

(c) 6%

(d) 20%

$$\text{Sol: } \frac{\delta x}{x} \times 100 = 5\%$$

$$\frac{\delta S}{S} \times 100 = \frac{12x}{6x^2} = \frac{\delta x}{x} \times 100$$

$$= 2 \times \frac{\delta x}{x} \times 100$$

$$= 2 \times 5 = 10\%$$

Ans: (a)

8. The value of $\int \frac{1+x^4}{1+x^6} dx$ is

(a) $\tan^{-1} x + \tan^{-1} x^3 + C$

(b) $\tan^{-1} x + \frac{1}{3} \tan^{-1} x^3 + C$

(c) $\tan^{-1} x - \frac{1}{3} \tan^{-1} x^3 + C$

(d) $\tan^{-1} x + \frac{1}{3} \tan^{-1} x^2 + C$

$$\text{Sol: } \int \frac{(x^4 - x^2 + 1) + x^2}{(x^2 + 1)(x^4 - x^2 + 1)} dx$$

$$= \int \frac{1}{x^2 + 1} dx + \frac{1}{3} \int \frac{3x^2}{x^6 + 1} dx$$

$$\text{Let } x^3 = t, 3x^2 dx = dt$$

$$= \tan^{-1} x + \frac{1}{3} \tan^{-1}(x^3) + C$$

Ans: (b)

9. The maximum value of $\frac{\log_e x}{x}$, if $x > 0$ is

(a) e

(b) 1

(c) $\frac{1}{e}$

(d) $-\frac{1}{e}$

$$\text{Sol: } f'(x) = \frac{1 - \log x}{x^2}$$

$$f'(x) = 0$$

$$x = e$$

$$f''(e) < 0$$

$$\therefore \text{on maximum value} = \frac{1}{e}$$

Ans: (c)

10. The value of $\int e^{\sin x} \sin 2x dx$ is

- | | |
|------------------------------------|------------------------------------|
| (a) $2e^{\sin x} (\sin x - 1) + C$ | (b) $2e^{\sin x} (\sin x + 1) + C$ |
| (c) $2e^{\sin x} (\cos x + 1) + C$ | (d) $2e^{\sin x} (\cos x - 1) + C$ |

Sol: Let $\sin x = t$

$$\cos x \, dx = dt$$

$$2 \int e^t t \, dt = 2e^t [t - 1] + C$$

$$= 2e^{\sin x} [\sin x - 1] + C$$

Ans: (a)

11. The value of $\int_{-1/2}^{1/2} \cos^{-1} x \, dx$ is

- | | | | |
|-----------|---------------------|-------|-----------------------|
| (a) π | (b) $\frac{\pi}{2}$ | (c) 1 | (d) $\frac{\pi^2}{2}$ |
|-----------|---------------------|-------|-----------------------|

$$\text{Sol: } I = \int_{-1/2}^{1/2} \cos^{-1}(x) \, dx \quad \dots (1)$$

$$I = \int_{-1/2}^{1/2} \cos^{-1}(-x) \, dx$$

$$I = \int_{-1/2}^{1/2} \pi - \cos^{-1} x \, dx \quad \dots (2)$$

$$(1) + (2)$$

$$2I = \int_{-1/2}^{1/2} \pi \, dx$$

$$2I = \pi (1)$$

$$I = \frac{\pi}{2}$$

Ans: (b)

12. If $\int \frac{3x+1}{(x-1)(x-2)(x-3)} \, dx = A \log|x-1| + B \log|x-2| + C \log|x-3| + C$, then the values of A, B and C

are respectively

- | | | | |
|---------------|---------------|--------------|--------------|
| (a) 5, -7, -5 | (b) 2, -7, -5 | (c) 5, -7, 5 | (d) 2, -7, 5 |
|---------------|---------------|--------------|--------------|

$$\text{Sol: } \frac{3x+1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$3x+1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

Taking $x = 1, 2, 3$

$$A = 2, B = -7, C = 5$$

Ans: (d)

13. The value of $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ is

(a) $\frac{\pi}{2} \log 2$

(b) $\frac{\pi}{4} \log 2$

(c) $\frac{1}{2}$

(d) $\frac{\pi}{8} \log 2$

Sol: Let $x = \tan \theta$

$$dx = \sec^2 \theta d\theta$$

$$I = \int_0^{\pi/4} \frac{\log(1 + \tan \theta)}{\sec^2 \theta} \times \sec^2 \theta d\theta$$

$$I = \int_0^{\pi/4} \log(1 + \tan \theta) d\theta \quad \dots (1)$$

$$I = \int_0^{\pi/4} \log \left(1 + \tan \left(\frac{\pi}{4} - \theta \right) \right) d\theta$$

$$= \int_0^{\pi/4} \log \left(\frac{2}{1 + \tan \theta} \right) d\theta$$

$$I = \log 2 \int_0^{\pi/4} 1 d\theta - I$$

$$2I = \log 2 \cdot \frac{\pi}{4}$$

$$I = \frac{\pi}{8} \log 2$$

Ans: (d)

14. The area of the region bounded by the curve $y^2 = 8x$ and the line $y = 2x$ is

(a) $\frac{16}{3}$ sq. units

(b) $\frac{4}{3}$ sq. units

(c) $\frac{3}{4}$ sq. units

(d) $\frac{8}{3}$ sq. units

$$\text{Sol: } A = \int_0^2 (2\sqrt{2}\sqrt{x} - 2x) dx$$

$$= \left(\frac{4\sqrt{2}}{3} x^{3/2} - x^2 \right) = \left(\frac{4\sqrt{2}}{3} 2^{3/2} - 4 \right) = \frac{4}{3}$$

Ans: (b)

15. The value of $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx$ is

(a) 2

(b) 0

(c) 1

(d) -2

$$\text{Sol: } I = \int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx \quad \dots (1)$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^{-x}} dx$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{e^x(\cos x)}{1+e^x} dx \quad \dots (2)$$

$$(1) + (2)$$

$$2I = \int_{-\pi/2}^{\pi/2} \cos x dx$$

$$= (\sin x)_{-\pi/2}^{\pi/2}$$

$$2I = 2$$

$$I = 1$$

Ans: (c)

16. The order of the differential equation obtained by eliminating arbitrary constants in the family of curves $c_1y = (c_2 + c_3)e^{x+c_4}$ is

(a) 1

(b) 2

(c) 3

(d) 4

$$\text{Sol: } c_1y = k_1e^x \cdot e^{cy}$$

$$y = \frac{k_1e^{cy}}{c_1} e^x$$

$$y = ke^x$$

order = 1

Ans: (a)

17. The general solution of the differential equation $x^2 dy - 2xydx = x^4 \cos x dx$ is

(a) $y = x^2 \sin x + cx^2$

(b) $y = x^2 \sin x + c$

(c) $y = \sin x + cx^2$

(d) $y = \cos x + cx^2$

$$\text{Sol: } \frac{dy}{dx} - \frac{2}{x}y = x^2 \cos x$$

$$\text{I.F.} = e^{\int \frac{-2}{x} dx} = \frac{1}{x^2}$$

$$\text{G.S } y \cdot \frac{1}{x^2} = \int \frac{1}{x^2} (x^2 \cos x) dx$$

$$\frac{y}{x^2} = \sin x + c$$

$$y = x^2 \sin x + cx^2$$

Ans: (a)

18. The area of the region bounded by the line $y = 2x + 1$, x -axis and the ordinates $x = -1$ and $x = 1$ is

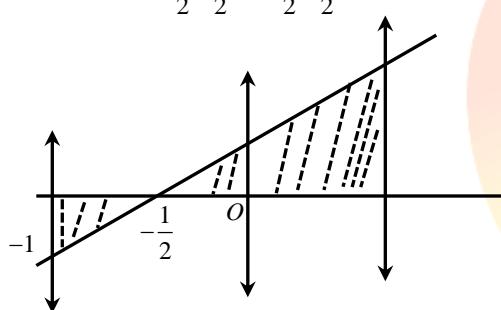
(a) $\frac{9}{4}$

(b) 2

(c) $\frac{5}{2}$

(d) 5

$$\text{Sol: Area} = \frac{1}{2} \times \frac{1}{2} \times 1 + \frac{1}{2} \times \frac{3}{2} \times 3$$



$$= \frac{1}{4} + \frac{9}{4} = \frac{5}{2}$$

Ans: (c)

19. The two vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} + 5\hat{k}$ represent the two sides \overrightarrow{AB} and \overrightarrow{AC} respectively of a $\triangle ABC$. The length of the median through A is

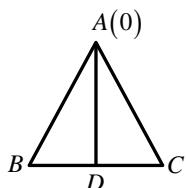
(a) $\frac{\sqrt{14}}{2}$

(b) 14

(c) 7

(d) $\sqrt{14}$

Sol: Consider A as origin



$$\overrightarrow{AD} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\text{Length of } AD = \sqrt{1+4+9} = \sqrt{14}$$

Ans: (d)

20. If \vec{a} and \vec{b} are unit vectors and θ is the angle between \vec{a} and \vec{b} , then $\sin \frac{\theta}{2}$ is

(a) $|\vec{a} + \vec{b}|$

(b) $\frac{|\vec{a} + \vec{b}|}{2}$

(c) $\frac{|\vec{a} - \vec{b}|}{2}$

(d) $|\vec{a} - \vec{b}|$

$$\text{Sol: } |\vec{a} - \vec{b}| = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$$

$$= 1 + 1 - 2\cos\theta$$

$$= 2(1 - \cos\theta)$$

$$|\vec{a} - \vec{b}|^2 = 2 \cdot 2\sin^2\left(\frac{\theta}{2}\right)$$

$$|\vec{a} - \vec{b}| = 2\sin\left(\frac{\theta}{2}\right)$$

Ans: (c)

21. The curve passing through the point $(1, 2)$ given that the slope of the tangent at any point (x, y)

is $\frac{3x}{y}$ represents

(a) Circle

(b) Parabola

(c) Ellipse

(d) Hyperbola

$$\text{Sol: } \frac{dy}{dx} = \frac{2x}{y}$$

$$ydy = 2xdx$$

$$\frac{y^2}{2} = x^2 + c \text{ passes through } (1, 2)$$

$$C = 1$$

$$\therefore \frac{y^2}{2} = x^2 + 1$$

$$\frac{x^2}{1} - \frac{y^2}{2} = -1$$

Represents hyperbola

Ans: (d)

22. If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$ and $|\vec{a}| = 6$ then $|\vec{b}|$ is equal to

(a) 6

(b) 3

(c) 2

(d) 4

$$\text{Sol: } |\vec{a}|^2 |\vec{b}|^2 = 144$$

$$|\vec{b}|^2 = 4$$

$$|\vec{b}| = 2$$

Ans: (c)

23. The point $(1, -3, 4)$ lies in the octant

(a) Second

(b) Third

(c) Fourth

(d) Eighth

Sol: Fourth octant

Ans: (c)

24. If the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$, $2\hat{i} + \hat{j} - \hat{k}$ and $\lambda\hat{i} - \hat{j} + 2\hat{k}$ are coplanar, then the value of λ is

(a) 6

(b) -5

(c) -6

(d) 5

Sol:
$$\begin{vmatrix} 2 & -3 & 4 \\ 2 & 1 & -1 \\ \lambda & -1 & 2 \end{vmatrix} = 0$$

$$2(2-1) + 3(4+\lambda) + 4(-2-\lambda) = 0$$

$$\lambda = 6$$

Ans: (a)

25. The distance of the point $(1, 2, -4)$ from the line $\frac{x-3}{2} = \frac{y-3}{3} = \frac{z+5}{6}$ is

(a) $\frac{293}{7}$

(b) $\frac{\sqrt{293}}{7}$

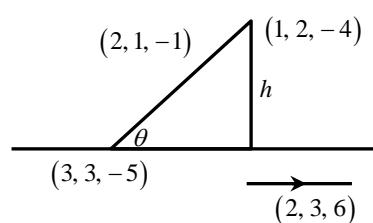
(c) $\frac{293}{49}$

(d) $\frac{\sqrt{293}}{49}$

Sol: $\cos\theta = \frac{4+3-6}{\sqrt{6} \times 7} = \frac{1}{7\sqrt{6}}$

$$\sin\theta = \frac{h}{\sqrt{6}}$$

$$h = \sqrt{6}\sqrt{1-\cos^2\theta} = \frac{\sqrt{293}}{7}$$



Ans: (b)

26. The sine of the angle between the straight line $\frac{x-2}{3} = \frac{3-y}{-4} = \frac{z-4}{5}$ and the plane $2x - 2y + z = 5$ is

(a) $\frac{3}{\sqrt{50}}$

(b) $\frac{3}{50}$

(c) $\frac{4}{5\sqrt{2}}$

(d) $\frac{\sqrt{2}}{10}$

Ans: (g)

Sol: $\vec{b}(3, -4, 5)$, $\vec{n} = (2, -2, 1)$

$$\sin\theta = \frac{\vec{b} \cdot \vec{n}}{|\vec{b}| |\vec{n}|} = \frac{1}{\sqrt{50}}$$

27. If a line makes an angle of $\frac{\pi}{3}$ with each of x and y -axis, then the acute angle made by z -axis is

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{2}$

Sol: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\frac{1}{4} + \frac{1}{4} + \cos^2 \gamma = 1$$

$$\cos^2 \gamma = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\cos \gamma = \frac{1}{\sqrt{2}}$$

$$\gamma = \frac{\pi}{4}$$

Ans: (a)

28. Corner points of the feasible region determined by the system of linear constraints are $(0, 3), (1, 1)$ and $(3, 0)$. Let $z = px + qy$, where $p, q > 0$. Condition on p and q so that the minimum of z occurs at $(3, 0)$ and $(1, 1)$ is

(a) $p = 2q$

(b) $p = \frac{q}{2}$

(c) $p = 3q$

(d) $p = q$

Sol: $z = px + qy$

Z occurs maximum at $(3, 0), (1, 1)$

$$3p = p + q$$

$$2p = q$$

Ans: (b)

29. The feasible region of an LPP is shown in the figure.

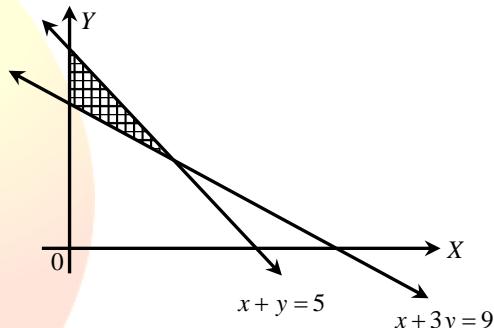
If $Z = 11x + 7y$, then the maximum value of Z occurs at

(a) $(0, 5)$

(b) $(3, 3)$

(c) $(5, 0)$

(d) $(3, 2)$



Sol: Corner points $(0, 3), (0, 5), (3, 2)$

\therefore maximum value at $(3, 2)$

Ans: (d)

30. A die is thrown 10 times, the probability that an odd number will come up atleast one time is

(a) $\frac{1}{1024}$

(b) $\frac{1023}{1024}$

(c) $\frac{11}{1024}$

(d) $\frac{1013}{1024}$

Sol: $n = 10, p = \frac{3}{6} = \frac{1}{2}, q = \frac{1}{2}$

$$p(x \geq 1) = 1 - p(x = 0)$$

$$= 1 - \left[10C_0 \left(\frac{1}{2} \right)^{10} \right] = 1 - \frac{1}{2^{10}}$$

$$= 1 - \frac{1}{1024} = \frac{1023}{1024}$$

Ans: (b)

31. If A and B are two events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{6}$, then $P\left(\frac{A'}{B}\right)$ is

(a) $\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$

(d) $\frac{1}{12}$

$$\text{Sol: } P\left(\frac{A'}{B}\right) = \frac{P(A' \cap B)}{P(B)}$$

$$= \frac{P(B) - P(A \cap B)}{P(B)}$$

$$= \left(\frac{1}{2} - \frac{1}{6} \right) 2$$

$$= 2 \left[\frac{3-1}{6} \right] = \frac{2}{3}$$

Ans: (a)

32. Events E_1 and E_2 from a partition of the sample space S . A is any event such that

$$P(E_1) = P(E_2) = \frac{1}{2}, P\left(\frac{E_2}{A}\right) = \frac{1}{2} \text{ and } P\left(\frac{A}{E_2}\right) = \frac{2}{3}, \text{ then } P\left(\frac{E_1}{A}\right) \text{ is}$$

(a) $\frac{1}{2}$

(b) $\frac{2}{3}$

(c) 1

(d) $\frac{1}{4}$

$$\text{Sol: } \frac{P(E_2 \cap A)}{P(A)} = \frac{1}{2}, \frac{P(A \cap E_2)}{P(E_2)} = \frac{2}{3}$$

$$P(A) = \frac{2}{3}, P(A \cap E_2) = \frac{1}{3}$$

$$A \cap (E_1 \cup E_2) = A$$

$$P(A \cap E_1) + P(A \cap E_2) = P(A)$$

$$P(A \cap E_1) = \frac{1}{3}$$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1 \cap A)}{P(A)} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

Ans: (a)

33. The probability of solving a problem by three persons A, B and C independently is $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{3}$ respectively. Then the probability of the problem is solved by any two of them is

(a) $\frac{1}{12}$

(b) $\frac{1}{4}$

(c) $\frac{1}{24}$

(d) $\frac{1}{8}$

Sol: $P(A) = \frac{1}{2}, P(B) = \frac{1}{4}, P(C) = \frac{1}{3}$

Probability of the problem solved by any two

$$= P(A)P(B)P(\bar{C}) + P(A)P(\bar{B})P(C) + P(\bar{A})P(B)P(C)$$

$$= \frac{1}{4}$$

Ans: (b)

34. If $n(A) = 2$ and total number of possible relations from set A to set B is 1024, then $n(B)$ is

(a) 512

(b) 20

(c) 10

(d) 5

Sol: $2 \times p = 10$

$$2 \times 1024 = 2^p$$

$$2^p = 10$$

$$n(B) = p = 5$$

Ans: (d)

35. The value of $\sin^2 51^\circ + \sin^2 39^\circ$ is

(a) 1

(b) 0

(c) $\sin 12^\circ$

(d) $\cos 12^\circ$

Sol: $\sin^2 51^\circ - \cos^2 39^\circ + 1$

$$\cos(51+39)\cos(51-39)+1$$

$$\cos 90 \times \cos 12 = 0 + 1$$

Ans: (a)

36. If $\tan A + \cot A = 2$, then the value of $\tan^4 A + \cot^4 A =$

(a) 2

(b) 1

(c) 4

(d) 5

Sol: $\tan A + \frac{1}{\tan A} = 2$

$$(\tan A - 1)^2 = 0$$

$$\tan A = 1, A = 45^\circ$$

$$\tan^4 45^\circ + \cot^4 45^\circ = 2$$

Ans: (a)

37. If $A = \{1, 2, 3, 4, 5, 6\}$, then the number of subsets of A which contain atleast two elements is

- (a) 64 (b) 63 (c) 57 (d) 58

Sol: $P(A) = 2^6 = 64$

$P(A) - 7 = 64 - 7 = 57$

Ans: (c)

38. If $z = x + iy$, then the equation $|z+1| = |z-1|$ represents

- (a) a circle (b) a parabola (c) x -axis (d) y -axis

Sol: $(x+1)^2 + y^2 = (x-1)^2 + y^2$

$4x = 0$

$x = 0$, y -axis

Ans: (d)

39. The value of ${}^{16}C_9 + {}^{16}C_{10} - {}^{16}C_6 - {}^{16}C_7$ is

- (a) 0 (b) 1 (c) ${}^{17}C_{10}$ (d) ${}^{17}C_3$

Sol: ${}^nC_r + {}^nC_{r-1} = {}^{(n+1)}C_r$

${}^{16}C_9 + {}^{16}C_{10} - ({}^{16}C_6 + {}^{16}C_7)$

${}^{17}C_{10} - {}^{17}C_7$

${}^{17}C_{17-10} - {}^{17}C_7 = 0 \quad (\because {}^nC_r = {}^nC_{n-r})$

Ans: (a)

40. The number of terms in the expansion of $(x+y+z)^{10}$ is

- (a) 66 (b) 142 (c) 11 (d) 110

Sol: ${}^{(n+r-1)}C_{r-1}$

$n = 10, r = 3$

$(10+3-1)C_{31} = {}^{12}C_2 = \frac{12 \times 11}{2} = 66$

Ans: (a)

41. If $P(n): 2^n < n!$. Then the smallest positive integer for which $P(n)$ is true if

- (a) 2 (b) 3 (c) 4 (d) 5

Sol: $2^n < 4! \Rightarrow 16 < 24$

Ans: (c)

42. The two lines $lx + my = n$ and $l'x + m'y = n'$ are perpendicular if

- (a) $ll' + mm' = 0$ (b) $lm' = ml'$ (c) $lm + l'm' = 0$ (d) $lm' + ml' = 0$

$$\text{Sol: } m_1 = \frac{-l}{m}, \quad m_2 = \frac{-l'}{m'}$$

$$m_1 \times m_2 = -1$$

$$\left(\frac{-l}{m}\right)\left(\frac{-l'}{m'}\right) = -1$$

$$ll' + mn' = 0$$

Ans: (a)

43. If the parabola $x^2 = 4ay$ passes through the point $(2, 1)$, then the length of the latus rectum is

- (a) 1 (b) 4 (c) 2 (d) 8

$$\text{Sol: } x^2 = 4ay \Rightarrow 2^2 = 4a \times 1 \Rightarrow a = 1$$

$$4a = 4 \times 1 = 4$$

Ans: (b)

44. If the sum of n terms of an A.P is given by $S_n = n^2 + n$, then the common difference of the A.P is

- (a) 4 (b) 1 (c) 2 (d) 6

$$\text{Sol: } d = S_2 - 2S_1$$

Ans: (c)

45. The negation of the statement "For all real numbers x and y , $x + y = y + x$ " is

- (a) For all real numbers x and y , $x + y \neq y + x$
 (b) For some real numbers x and y , $x + y = y + x$
 (c) For some real numbers x and y , $x + y \neq y + x$
 (d) For some real numbers x and y , $x - y = y - x$

Sol: For some real numbers x and y , $x + y \neq y + x$

Ans: (c)

46. The standard deviation of the data $6, 7, 8, 9, 10$ is

- (a) $\sqrt{2}$ (b) $\sqrt{10}$ (c) 2 (d) 10

$$\text{Sol: } 6, 7, 8, 9, 10, \quad \bar{x} = 8, \quad n = 5$$

$$\text{S.D.} = \sqrt{\frac{1}{n} \left(x_1^2 \right) - \bar{x}^2} = \sqrt{66 - 64} = \sqrt{2}$$

Ans: (a)

47. $\lim_{x \rightarrow 0} \left(\frac{\tan x}{\sqrt{2x+4} - 2} \right)$ is equal to

(a) 2

(b) 3

(c) 4

(d) 6

$$\text{Sol: } \lim_{x \rightarrow 0} \frac{\tan x (\sqrt{2x+4} + 2)}{2x+4 - 4}$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} \times \lim_{x \rightarrow 0} \frac{(\sqrt{2x+4} + 2)}{2}$$

$$1 \times \frac{2+2}{2} = 2$$

Ans: (a)

48. If a relation R on the set $\{1, 2, 3\}$ be defined by $R = \{(1, 1)\}$, then R is

(a) Reflexive and symmetric

(b) Reflexive and transitive

(c) Symmetric and transitive

(d) Only symmetric

Sol: Symmetric and transitive

Ans: (c)

49. Let $f : [2, \infty) \rightarrow R$ be the function defined $f(x) = x^2 - 4x + 5$, then the range of f is

(a) $(-\infty, \infty)$

(b) $[1, \infty)$

(c) $(1, \infty)$

(d) $[5, \infty)$

$$\text{Sol: } y = f(x) = x^2 - 4x + 5 = (x-2)^2 + 1$$

$$1 + (x-2)^2 \geq 0 + 1$$

$$y \geq 1 \Rightarrow y \in [1, \infty)$$

Ans: (b)

50. If A, B, C are three mutually exclusive and exhaustive events of an experiment such that

$P(A) = 2P(B) = 3P(C)$, then $P(B)$ is equal to

(a) $\frac{1}{11}$

(b) $\frac{2}{11}$

(c) $\frac{3}{11}$

(d) $\frac{4}{11}$

$$\text{Sol: } \frac{P(A)}{6} = \frac{2P(B)}{6} = \frac{3P(C)}{6}$$

$$\frac{P(A)}{6} = \frac{P(B)}{3} = \frac{P(C)}{2} = K$$

$$P(A) = 6K, P(B) = 3K, P(C) = 2K$$

$$P(A) + P(B) + P(C) = 1 \Rightarrow 11K = 1, K = \frac{1}{11}$$

Ans: (c)

51. The domain of the function defined by $f(x) = \cos^{-1} \sqrt{x-1}$ is

(a) $[1, 2]$

(b) $[0, 2]$

(c) $[-1, 1]$

(d) $[0, 1]$

Sol: $0 \leq \sqrt{x-1} \leq 1$

$$0 \leq (x-1) \leq 1$$

$$1 \leq x \leq 1+1 \Rightarrow [1, 2]$$

Ans: (a)

52. The value of $\cos\left(\sin^{-1} \frac{\pi}{3} + \cos^{-1} \frac{\pi}{3}\right)$ is

(a) 0

(b) 1

(c) -0

(d) Does not exist

Sol: $\cos\left(\frac{\pi}{2}\right) = 0$ $\left(\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}\right)$

Ans: (a)

53. If $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, then A^4 is equal to

(a) A

(b) $2A$

(c) I

(d) $4A$

Sol: $A \cdot A = A^2 = I$

$$A^4 = A^2 \cdot A^2 = I \cdot I = I$$

$$A^4 = I$$

Ans: (c)

54. If $A = \{a, b, c\}$, then the number of binary operations on A is

(a) 3

(b) 3^6

(c) 3^3

(d) 3^9

Sol: By Conception. n^{n^2}

$$= 3^{3^2} = 3^9$$

Ans: (d)

55. If $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then the matrix a is

(a) $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} -2 & 1 \\ 3 & -2 \end{pmatrix}$

(d) $\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$

Sol: $BA = I \Rightarrow B = A^{-1}$

Ans: (b)

56. If $f(x) = \begin{vmatrix} x^3 - x & a+x & b+x \\ x-a & x^2 - x & c+x \\ x-b & x-c & 0 \end{vmatrix}$ then

- (a) $f(1) = 0$ (b) $f(2) = 0$ (c) $f(0) = 0$ (d) $f(-1) = 0$

Sol: By substitution

Ans: (c)

57. If A and B are square matrices of same order and B is a skew symmetric matrix, then $A'BA$ is

- | | |
|----------------------|---------------------------|
| (a) Symmetric matrix | (b) Null matrix |
| (c) Diagonal matrix | (d) Skew symmetric matrix |

Sol: $(A'BA)^+ = (BA)^+ (A')^+ B^+ = -3$

$$= A'B^T A$$

$$= A'(-B)A$$

$$= -A'BA$$

Ans: (d)

58. If A is a square matrix of order 3 and $|A| = 5$, then $|A \ adj \cdot A|$ is

- (a) 5 (b) 125 (c) 25 (d) 625

Sol: $(\text{Adj}) = |A|^n = 5^3 = 125$

Ans: (b)

59. If $f(x) = \begin{cases} \frac{1-\cos Kx}{x \sin x}, & \text{If } x \neq 0 \\ \frac{1}{2}, & \text{If } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of K is

- (a) $\pm \frac{1}{2}$ (b) 0 (c) ± 2 (d) ± 1

Sol: $\lim_{x \rightarrow 0} \frac{1-\cos|x|}{x \sin x}, f(0) = \frac{1}{2}$ by L'Hopital rule

$$\frac{k \sin x}{x \cos x + \sin x} = \lim_{x \rightarrow 0} \frac{k^2 \cos x}{-x \sin x + \cos x + \cos x}$$

$$= \frac{k^2 \times 1}{1+1} = \frac{1}{2}$$

$$k = \pm 1$$

Ans: (d)

60. If $a_1, a_2, a_3, \dots, a_9$ are in A.P. then the value of $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is
- (a) $\frac{9}{2}(a_1 + a_9)$ (b) $a_1 + a_9$ (c) $\log_e(\log_e e)$ (d) 1

Sol: By verification

$$8, 1, 2, \dots, 9, \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} 20$$

$$\log(\log e) = \log 1 = 0$$

Ans: (c)



Key Answers:

1. b	2. b	3. b	4. b	5. d	6. a	7. a	8. b	9. c	10. a
11. b	12. d	13. d	14. b	15. c	16. a	17. a	18. c	19. d	20. c
21. d	22. c	23. c	24. a	25. b	26.	27. a	28. b	29. d	30. b
31. a	32. a	33. b	34. d	35. a	36. a	37. c	38. d	39. a	40. a
41. c	42. a	43. b	44. c	45. c	46. a	47. a	48. c	49. b	50. c
51. a	52. a	53. c	54. d	55. b	56. c	57. d	58. b	59. d	60. c

26. (g)

