KCET Board Exam – 2022

Subject: Mathematics

1. The solution of the following equation $\frac{dy}{dx} = (x+y)^2$ is

(A) $\cot^{-1}(x+y) = x+c$ (B) $\tan^{-1}(x+y) = x+c$

(C) $\tan^{-1}(x+y) = 0$ (D) $\cot^{-1}(x+y) = c$

Sol: $\frac{dy}{dx} = x^2 + y^2 + 2xy$

Option verification

$$\frac{1}{1+(x+y)^2}\left(1+\frac{dy}{dx}\right) = 1$$

$$1+\frac{dy}{dx} = 1+(x+y)^2$$

$$\frac{dy}{dx} = (x+y)^2$$

Ans: (B)

2. If $y(x)$ be the solution of differential equation $x\log x\frac{dy}{dx} + y = 2x\log x,\ y(e)$ is equal to

(A) $2e$ (B) e (C) 0 (D) 2

Sol: $x\log x\frac{dy}{dx} + y = 2x\log x$ … (1)

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x\log x} = 2$$

I.F. $= e^{\int \frac{1}{x\log x}dx} = e^{\log(\log x)}$

$y\times\log x = \int 2\cdot\log x\ dx = 2[x\log x - x] + C$ … (2)

$$y = \frac{2[x\log x - x]}{\log x} + \frac{C}{\log x}$$

$y(e) = 0 + \frac{C}{1} = C$ … (3)

From Eqn (1) for $x = 1,\ y = 0$

$\therefore\ y(1) = 0$

From Eqn (2), $0 = 2[1.0 - 1] + C$

$\Rightarrow C = +2$

From Eqn (3), $y(e) = 2$

Ans: (D)

3. If $|\vec{a}| = 2$ and $|\vec{b}| = 3$ and the angle between $\vec{a}$ and $\vec{b}$ is $120°$, then the length of the vector $\left|\frac{1^{\vec{a}}}{2} - \frac{1^{\vec{b}}}{3}\right|^2$ is

(A) 1 (B) 2 (C) 3 (D) $\frac{1}{6}$

Sol: $\theta = 120°$

$\left(\frac{1}{2}\bar{a}\right)^2 + \left(\frac{1}{3}\vec{b}\right)^2 - 2.\frac{1}{2}.\frac{1}{3}\bar{a}\cdot\vec{b}$

$= \frac{1}{4}.4 + \frac{1}{9}.9 - \frac{1}{3}.2.3.\cos 120$

$= 1 + 1 - 2\cos(90 + 30)$

$= 2 - 2\cdot(-\sin 30°)$

$= 2 + 2\cdot\frac{1}{2} = 3$

Ans: (C)

4. If $|\vec{a}\times\vec{b}| + |\vec{a}.\vec{b}|^2 = 36$ and $|\vec{a}| = 3$ then $|\vec{b}|$ is equal to

(A) 2 (B) 9 (C) 36 (D) 4

Sol: $|\vec{a}\times\vec{b}| + |\vec{a}\cdot\vec{b}|^2 = 36 \Rightarrow |\vec{a}|^2 |\vec{b}|^2 = 36$

$\Rightarrow 3^2 \times |\vec{b}|^2 = 3^2 \times 2^2 \Rightarrow |\vec{b}| = 2$

Ans: (A)

5. If $\vec{\alpha} = \hat{i} - 3\hat{j}, \vec{\beta} = \hat{i} + 2\hat{j} - \hat{k}$ then express $\vec{\beta}$ in the form $\vec{\beta} = \overrightarrow{\beta_1} + \overrightarrow{\beta_2}$ where $\overrightarrow{\beta_1}$ is parallel to $\vec{\alpha}$ and $\overrightarrow{\beta_2}$ is perpendicular to $\vec{\alpha}$ then $\overrightarrow{\beta_1}$ is given by

(A) $\hat{i} + 3\hat{j}$ (B) $\frac{5}{8}(\hat{i} - 3\hat{j})$ (C) $\frac{5}{8}(\hat{i} + 3\hat{j})$ (D) $\hat{i} - 3\hat{j}$

Sol: $\vec{\alpha} = \hat{i} - 3\hat{j},\ \beta = \hat{i} + 2\hat{j} - \hat{k}$

β_1 is parallel to α

$\therefore\ \beta_1 = k_1 \cdot \alpha = k_1\hat{i} - 3k_1\hat{j}$

$\beta_2 = \beta - \beta_1 = (1 - k_1)\hat{i} + (2 + 3k_1)\hat{j} - \hat{k}$

$\alpha \cdot \beta_2 = 0 \Rightarrow 1 - k_1 - 3(2 + 3k_1) = 0$

$\Rightarrow 1 - 6 - k_1 - 9k_1 = 0 \Rightarrow k_1 = -\frac{1}{2}$

$\therefore\ \beta_1 = -\frac{1}{2}(\hat{i} - 3\hat{j}) = -\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j}$

$\beta_2 = \frac{3}{2}\hat{i} + \frac{1}{2}\hat{j} - \hat{k}$

Ans: (None of the options are correct)

6. The sum of the degree and order of the differential equation $\left(1+{y_1}^2\right)^{2/3}=y_2$ is

(A) 7 (B) 4 (C) 6 (D) 5

Sol: $\left(1+y_1^2\right)^{2/3}=y_2$

$\left(1+y_1^2\right)=y_2^3$

$\text{Order}=2$

$\text{Degree }=3$

$\text{Sum}=2+3=5$

Ans: (D)

7. If $\frac{dy}{dx}+\frac{y}{x}=x^2$, then $2y(2)-y(1)=$

(A) $\frac{13}{4}$ (B) $\frac{11}{4}$ (C) $\frac{15}{4}$ (D) $\frac{9}{4}$

Sol: $P(x)=\frac{1}{x}$ $Q(x)=x^2$

$$I\cdot F=e^{\int\frac{1}{x}dx}=e^{\log x}=x$$

$$y\cdot x=\int x\cdot x^2dx$$

$$y\cdot x=\frac{x^4}{4}+C$$

$$y=\frac{x^3}{4}+\frac{C}{x}$$

$$y(1)=\frac{1}{4}+C \qquad y(2)=\frac{8}{4}+\frac{C}{2}$$

$$2y(2)-y(1)$$

$$2\left(2+\frac{C}{2}\right)-\frac{1}{4}-C$$

$$4+C-\frac{1}{4}-C$$

$$=\frac{15}{4}$$

Ans: (C)

8. The angle between the pair of lines $\frac{x+3}{3}=\frac{y-1}{5}=\frac{z+3}{4}$ and $\frac{x+1}{1}=\frac{y-4}{4}=\frac{z-5}{2}$ is

(A) $\theta=\cos^{-1}\left[\frac{5\sqrt{3}}{16}\right]$ (B) $\theta=\cos^{-1}\left[\frac{27}{5}\right]$ (C) $\theta=\cos^{-1}\left[\frac{8\sqrt{3}}{15}\right]$ (D) $\theta=\cos^{-1}\left[\frac{19}{21}\right]$

Sol: $\cos\theta=\frac{3+20+8}{\sqrt{9+25+16}\cdot\sqrt{1+16+4}}=\frac{31}{\sqrt{50}\sqrt{21}}=\frac{31}{5\sqrt{2}\sqrt{21}}$

Ans: (none of the options is correct)

9. The corner points of the feasible region of an LPP are $(0,2),(3,0),(6,0),(6,8)$ and $(0,5)$, then the minimum value of $z=4x+6y$ occurs at

(A) Only two points (B) Finite number of points

(C) Infinite number of points (D) Only one point

Sol: $z=4x+6y$

$(0,\ 2)$ $z=12$

$(3,\ 0)$ $z=12$

$\therefore$ z is min at only two points

Ans: (A)

10. A dietician has to develop a special diet using two foods X and Y. Each packet (containing 30g) of food. X contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Y contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atlaest 240 units of calcium, atleast 460 units iron and atmost 300 units of cholesterol. The corner points of the feasible region are

(A) $(2,72),(40,15),(115,0)$ (B) $(2,72),(40,15),(15,20)$

(C) $(2,72),(15,20),(0,23)$ (D) $(0,23),(40,15),(2,72)$

Sol: $x\ \frac{cal}{12}\quad \frac{Iron}{4}\quad \frac{Cuo}{6}\quad \frac{VitA}{6}$

y 3 20 4 3

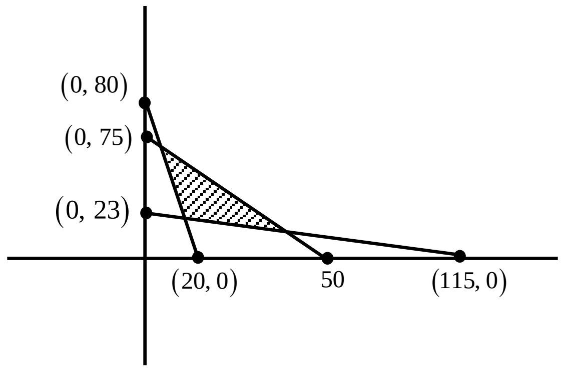
$\geq 240\quad \geq 460\quad \leq 300$

$z=6x+3y$

$12x+3y\geq 240\quad 4x+20y\geq 460\quad 6x+4y\leq 300$

$4x+y\geq 80$ … (1) $x+5y\geq 115$ … (2)

$3x+2y\leq 150$



$4x+y=80$

$x+5y=115$

$3x+24y=150$

$4x+y=80\rightarrow\times 5$

$x+5y=115$

$\Rightarrow$ $20x+5y=400$

$x+5y=115$

$19x=285$

$x=\frac{285}{19}=15$

$60+4y=80$

$4y=20$

$(15,\ 20)$

$x+54=115\times 3 \quad \Rightarrow \; 3x+15y=345$

$x+75=115 \qquad 3x+2y=150$

$x=40 \qquad 13y=195$

$(40,15) \qquad y=15$

Ans: (B)

11. The distance of the point whose position vector is $\left(2\hat{i}+\hat{j}-\hat{k}\right)$ from the plane $\vec{r}.\left(\hat{i}-2\hat{j}+4\hat{k}\right)=4$ is

(A) $\frac{-8}{21}$ (B) $\frac{8}{\sqrt{21}}$ (C) $8\sqrt{21}$ (D) $\frac{-8}{\sqrt{21}}$

Sol: $(2,1,-1)$

$\overline{N}=\hat{i}-2\hat{j}+4\hat{k}$

$\frac{\left|\vec{a}\cdot\overline{N}\cdot-d\right|}{\left|\overline{N}\right|}$

$=\frac{|2-2-4-4|}{\sqrt{1+4+16}}$

$=\frac{|-4-4|}{\sqrt{21}}=\frac{|-8|}{\sqrt{21}}=\frac{8}{\sqrt{21}}$

Ans: (B)

12. The co-ordination of foot of the perpendicular drawn from the origin to the plane $2x-3y+4z=29$ are

(A) $(-2,-3,4)$ (B) $(2,3,4)$ (C) $(2,-3,-4)$ (D) $(2,-3,4)$

Sol: $2x-3y+4z=29$

By $\sqrt{4+9+16}=\sqrt{29}$

$l=\frac{2}{\sqrt{29}}, m=\frac{-3}{\sqrt{29}} \qquad n=\frac{y}{\sqrt{29}} d=\frac{29}{\sqrt{29}}$

Root $\delta \quad x \quad y'$

(ld, md, nd)

$=(2,-3,4)$

Ans: (D)

13. If A and B are two events such that $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}$ and $P(A/B)=\frac{1}{4}$, then $P\left(A^{'}\cap B^{'}\right)$ is

(A) $\frac{3}{4}$ (B) $\frac{1}{4}$ (C) $\frac{3}{16}$ (D) $\frac{1}{12}$

Sol: $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}, P(A/B)=\frac{1}{4}$

$\frac{P(A\cap B)}{P(B)}=\frac{1}{4}\Rightarrow P(A\cap B)=\frac{1}{4}\times\frac{1}{3}=\frac{1}{12}$

$P(A'\cap B')=1-P(A\cup B)$

$=1-\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{12}\right)=1-\left(\frac{6+4-1}{12}\right)$

$=1-\frac{9}{12}=1-\frac{3}{4}=\frac{1}{4}$

Ans: (B)

14. A pandemic has been spreading all over the world. The probabilities are 0.7 that there will be a lockdown, 0.8 that the pandemic is controlled in one month if there is a lockdown and 0.3 that it is controlled in one month if there is no lockdown. The probability that the pandemic will be controlled in one month is

(A) 0.46 (B) 0.65 (C) 1.65 (D) 1.46

Sol:

Prob(Lock down) $=0.7$ Prob(No. Lock down) $=0.3$

Prob(controlled 1M) $=0.8$ Prob(controlled 1M) $=0.3$

$P(1\text{M})=0.7\times0.8+0.3\times0.3$

$=0.56+0.09$

$=0.65$

Ans: (B)

15. If A and B are two independent events such that $P(\bar{A})=0.75, P(A\cup B)=0.65,$ and $P(B)=x,$ then find the value of x:

(A) $\frac{7}{15}$ (B) $\frac{5}{14}$ (C) $\frac{8}{15}$ (D) $\frac{9}{14}$

Sol: $0.65=0.25+x-x(0.25)$

$0.4=x(0.75)\Rightarrow x=\frac{0.4}{0.75}=\frac{40}{75}=\frac{8}{15}$

Ans: (C)

16. Find the mean number of heads in three tosses of a fair coin:

(A) 3.5 (B) 1.5 (C) 4.5 (D) 2.5

Sol: X= No. of heads

$X=$ 0 1 2 3

P 1/8 3/8 3/8 1/8

Mean $=0.\frac{1}{8}+1.\frac{3}{8}+2.\frac{3}{8}+3.\frac{1}{8}$

$=\frac{3+6+3}{8}=\frac{12}{8}=\frac{3}{2}=1.5$

Ans: (B)

17. The domain of the function $f(x)=\frac{1}{\log_{10}(1-x)}+\sqrt{x+2}$ is

(A) $[-2,0)\cup(0,1)$ (B) $[-2,0)\cap(0,1)$ (C) $[-2,1)$ (D) $[-2,0)$

Sol:

$1-x>0 \Rightarrow x<1$ also $x \neq 0$

$x+2 \geq 0 \Rightarrow x \geq -2$

$\Rightarrow x \in [-2,0) \cup (0,1)$

Ans: (A)

18. The trigonometric function $y=\tan x$ in the II quadrant

(A)increases from $-\infty$ to 0 (B)decreases from 0 to ∞

(C)decreases from $-\infty$ to 0 (D)increases from 0 to ∞

Sol:

Increases from $-\infty$ to 0

Ans: (A)

19. The degree measure of $\frac{\pi}{32}$ is equal to

(A) $4^{\circ}30'30''$ (B) $5^{0}30'20''$ (C) $5^{0}37'20''$ (D) $5^{0}37'30''$

Sol:

$\left(\frac{\pi}{32}\right)^{c} = \left(\frac{\pi}{32} \times \frac{180}{\pi}\right)^{0} = \left(\frac{45}{8}\right)^{0} = 5^{0}\,37'30''$

Ans: (D)

20. The value of $\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$ is

(A) $\frac{1}{4}$ (B)0 (C)1 (D) $\frac{1}{2}$

Sol: $\sin 75^{\circ}\sin 15^{\circ} = \cos 15^{\circ}\sin 15^{\circ}$

$= \frac{1}{2}\sin 30^{\circ} = \frac{1}{4}$

Ans: (A)

21. $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} =$

(A) $2\cos\frac{\theta}{2}$ (B) $\sin 2\theta$ (C) $2\cos\theta$ (D) $2\sin\theta$

Sol: $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}$

$= \sqrt{2+\sqrt{2+2\cos 4\theta}}$

$= \sqrt{2+2\cos 2\theta} = 2\cos\theta$

Ans: (C)

22. If $A=\{1,2,3,......10\}$ then number of subsets of A containing only odd numbers is

(A) 30 (B) 31 (C) 27 (D) 32

Sol: No. of subsets with odd number

= No. of subsets by $\{1, 3, 5, 7, 9\} = 2^{5} = 32$

Ans: (D)

23. Suppose that the number of elements in set A is p, the number of elements in set B is q and the number of elements in $A \times B$ is 7 then $p^2 + q^2 =$ _________.

(A) 49 (B) 50 (C) 51 (D) 42

Sol: $n(A) = p,\ n(B) = q$

$n(A \times B) = pq = 7 \Rightarrow p = 1, q = 7$ or $p = 7, q = 1$

$\therefore\ p^2 + q^2 = 49 + 1 = 50$

Ans: (B)

24. If $a_1, a_2, a_3, ...a_{10}$ is a geometric progression and $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals

(A) $2\left(5^2\right)$ (B) $3\left(5^2\right)$ (C) 5^4 (D) 5^3

Sol: $\frac{a_3}{a_1} = \frac{ar^2}{a} = 25$

$r = 5$

$\frac{a_9}{a_5} = \frac{ar^8}{ar^4} = r^4 = 5^4$

Ans: (C)

25. If the straight line $2x - 3y + 17 = 0$ is perpendicular to the line passing through the points $(7,17)$ and $(15, \beta)$, then β equals

(A) -29 (B) -5 (C) 5 (D) 29

Sol: $m_1 = \frac{2}{3}$ $m_2 = \frac{\beta - 17}{8}$

$m_1 \cdot m_2 = -1$

$\frac{\beta - 17}{8} \times \frac{2}{3} = -1 \Rightarrow \beta - 17 = -12 \Rightarrow \beta = -12 + 17 = 5$

$\beta = 5$

Ans: (C)

26. The octant in which the point $(2, -4, -7)$ lies is

(A)Fifth (B)Eighth (C)Third (D)Fourth

Sol: Eighth

Ans: (B)

27. If $f(x) = \begin{cases} x^2 - 1, & 0 < x < 2 \\ 2x + 3, & 2 \le x < 3 \end{cases}$, the quadratic equation whose roots are $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$ is

(A) $x^2 - 7x + 8 = 0$ (B) $x^2 - 14x + 49 = 0$

(C) $x^2 - 10x + 21 = 0$ (D) $x^2 - 6x + 9 = 0$

Sol: $\alpha = \lim_{x \to 2} x^2 - 1 = 4 - 1 = 3$

$\beta = \lim_{x \to 2} 2x + 3 = 7$

$x^2 - 10x + 21 = 0$

Ans: (C)

28. If $3x + i(4x - y) = 6 - i$ where x and y are real numbers, then the values of x and y are respectively,

(A) $3, 4$ (B) $3, 9$ (C) $2, 4$ (D) $2, 9$

Sol: $3x = 6 \quad 4x - y = -1$

$x = 2 \qquad 8 - y = -1$

$y = 9$

Ans: (D)

29. If all permutations of the letters of the word MASK are arranged in the order as in dictionary with or without meaning, which one of the following is 19^{th} word?

(A) AMSK (B)KAMS (C)SAMK (D)AKMS

Sol:

A, K, M, S

$A------- = 3! = 6$

$K------- = 6$

$M------- = \frac{6}{18}$

S, A, K, M

Ans: (None of the options is correct)

30. If the set x contains 7 elements and set y contains 8 elements, then the number of bijections from x to y is

(A) $8!$ (B) 0 (C) $8\ P_7$ (D) $7!$

Sol:

$n(x) = 7$

$n(y) = 8$

No of bisectors $= 0$

Ans: (B)

31. If f: $R \to R$ be defined

$$f(x) = \begin{cases} 2x & : \quad x > 3 \\ x^2 & : \quad 1 < x \le 3 \\ 3x & : \quad x \le 1 \end{cases}$$

Then $f(-1) + f(2) + f(4)$ is

(A)14 (B)5 (C)10 (D)9

Sol: $f(-1) = -3$

$f(2) = 4$

$f(4) = 8$

$f(-1) + f(2) + 6(9) - 3 + 4 + 8 = 9$

Ans: (D)

32. Let the relation R is defined in N by a Rb, if $3a+2b=27$ then R is

(A) $\{(2,1)(9,3)(6,5)(3,7)\}$ (B) $\{(1,12)(3,9)(5,6)(7,3)\}$

(C) $\left\{\left(0,\frac{27}{2}\right)(1,12)(3,9)(5,6)(7,3)\right\}$ (D) $\{(1,12)(3,9)(5,6)(7,3)(9,0)\}$

Sol: $3a=27-2b$

$b=\frac{27-3a}{2}$

$\{(1,12),(3,9),(5,6),(7,3)\}$

Ans: (B)

33. $\lim_{y\to 0}\frac{\sqrt{3+y^3}-\sqrt{3}}{y^3}=$

(A) $3\sqrt{2}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{3\sqrt{2}}$ (D) $2\sqrt{3}$

Sol: $\lim_{y\to 0}\frac{\frac{1}{2\sqrt{3+y^3}}3y^2-0}{3y^2}$

$=\frac{1}{2\sqrt{3}}$

Ans: (b)

34. If the standard deviation of the numbers $-1,0,1,k$ is $\sqrt{5}$ where $k>0$, then k is equal to

(A) $2\sqrt{6}$ (B) $4\sqrt{\frac{5}{3}}$ (C) $\sqrt{6}$ (D) $2\sqrt{\frac{10}{3}}$

Sol: $S.D=\sigma=\sqrt{5}$

$\sigma^2=5$

$\frac{\Sigma x_1^2}{n}-\left(\frac{\Sigma x_1^1}{n}\right)^2=5$

$\frac{1+0+1+k^2}{4}-\left(\frac{k}{4}\right)^2=5$

$\frac{2}{4}+\frac{k^2}{4}-\frac{k^2}{16}=5$

$\frac{4k^2-k^2}{16}=5-\frac{1}{2}$

$\frac{3k^2}{16}=\frac{9}{2}$

$k^2=24$

$k=\sqrt{6\times 4}$

$=2\sqrt{6}$

Ans: (A)

35. If $A=\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$, then the inverse of the matrix A^3 is

(A) $-A$ (B) A (C) -1 (D) 1

Sol: $A^2=\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^3=IA=A$

Inverse of $A^3=$ Inverse of A

$\Rightarrow A$

Ans: (B)

36. If A is a skew symmetric matrix, then A^{2021} is

(A)Skew symmetric matrix (B)Row matrix

(C)Column matrix (D)Symmetric matrix

Sol: $A^T=-A$

$\left(A^n\right)^T=(-1)^n A^n$

$A^n=(-1)^n\left(A^n\right)^T$

$n=\text{odd}$

Ans: (A)

37. If $A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ then $(aI+bA)^n$ is (where I is the identity matrix of order 2)

(A) a^nI+b^nA (B) $a^2I+a^{n-1}b\cdot A$ (C) $a^nI+n\cdot a^{n-1}b\cdot A$ (D) a^nI+na^nbA

Sol: $A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\Rightarrow A^n=0$

$aI+bA=\begin{bmatrix} a & 0 \\ o & a \end{bmatrix}+\begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$, $(aI+bA)^2=\begin{bmatrix} a & b \\ 0 & a \end{bmatrix}\cdot\begin{bmatrix} a & b \\ 0 & a \end{bmatrix}=\begin{bmatrix} a^2 & 2ab \\ 0 & a^2 \end{bmatrix}$

$(aI+bA)^2=aI^2+2abA=\begin{bmatrix} a^2 & 2ab \\ 0 & a^2 \end{bmatrix}$

$(aI+bA)^3=a^3I+3a^2bA+3abc=\begin{bmatrix} a^3 & 3a^2b \\ 0 & a^3 \end{bmatrix}$

Verification: $(aI+bA)^n=a^n\cdot I+na^{n-1}\cdot b\cdot A$

Ans: (C)

38. If A is a 3×3 matrix such that $|5\cdot adj\,A|=5$ then $|A|$ is equal to

(A) ± 5 (B) ± 1 (C) $\pm 1/25$ (D) $\pm 1/5$

Sol: $|5\cdot AdjA|=5$

$\Rightarrow 5^3|AdjA|=5$

$\Rightarrow\ 5^3 \cdot |A|^2 = 5$

$\Rightarrow\ |A|^2 = \frac{1}{5^2}$

$\Rightarrow\ |A| = \pm\frac{1}{5}$

Ans: (D)

39. If there are two values of $'a'$ which makes determinant

$$\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix}$$

Then the sum of these numbers is

(A)5 (B)-4 (C)9 (D)4

Sol: $\begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$

$\Rightarrow 1\left(2a^2+4\right)+2(4a+0)+5(8)=86$

$\Rightarrow 2a^2+4+8a+0+40=86$

$\Rightarrow 2a^2+8a+44=86$

$\Rightarrow 2a^2+8a-42=0$

$\Rightarrow a^2+4a-21=0$

$\Rightarrow a^2+7a-3a-21=0$

$a=-7 \qquad a=3$

$\therefore$sum=-7+3=-4

Ans: (B)

40. If the vertices of a triangle are $(-2,6)(3,-6)$ and $(1,5)$, then the area of the triangle is

(A) $35\,sq.units$ (B) $40\,sq.units$ (C) $15.5\,sq.units$ (D) $30\,sq.units$

Sol: $\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ -2 & 3 & 1 \\ 6 & -6 & 5 \end{vmatrix}$

$\Rightarrow \frac{1}{2}\{(15+6)-(-10-6)+(12-18)\} \Rightarrow \frac{1}{2}\{(21)+16-6\} \Rightarrow \frac{1}{2}\{31\} = 15.5\,sq$

Ans: (C)

41. Domain of $\cos^{-1}[x]$ is, where $[\ ]$ denotes a greatest integer function

(A) $[-1, 2)$ (B) $(-1, 2]$ (C) $(-1, 2)$ (D) $[-1, 2]$

Sol: $\cos^{-1}[x]$ is defined $-1 \le [x] \le 1$

$x \in [-1, 2)$

Ans: (A)

42. If A is matrix of order 3×3, then $\left(A^2\right)^{-1}$ is equal to

(A) $(-A)^{-2}$ (B) $\left(-A^2\right)^2$ (C) $\left(A^{-1}\right)^2$ (D) A^2

Sol: $\left(A^2\right)^{-1}=\frac{Adj\left(A^2\right)}{\left|A^2\right|}=\frac{(AdjA)^2}{\left|A^2\right|}$

$=\left(\frac{AdjA}{|A|}\right)^2=\left(A^{-1}\right)^2$

Ans: (C)

43. If $x=e^\theta\sin\theta,\ y=e^\theta\cos\theta$ where θ is a parameter, then $\frac{dy}{dx}$ at $(1,1)$ is equal to

(A) $-\frac{1}{4}$ (B) 0 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Sol: $\frac{dx}{d\theta}=e^\theta\cos\theta+e^\theta\sin\theta$

$\frac{dy}{d\theta}=e^\theta\cos\theta-e^\theta\sin\theta$

$\frac{dy}{dx}=\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$

$\Rightarrow 0$

Ans: (B)

44. If $y=e^{\sqrt{x\sqrt{x\sqrt{x}}}\,\ldots\ldots}\ x>1$ then $\frac{d^2y}{dx^2}$ at $x=\log_e^3$ is

(A) 1 (B) 3 (C) 5 (D) 0

Sol: $y=e^{\sqrt{x\sqrt{x\sqrt{x}}}\,\ldots}\Rightarrow\log_e^y=\sqrt{x\sqrt{x\sqrt{x}}}$

$\log_e^y=\sqrt{x\log_e^y}$

$\left(\log_e^y\right)^2=x\log_e^y$

$\log_e^y=x\Rightarrow y=e^x$

$\frac{dy}{dx}=e^x$

$\frac{d^2y}{dx^2}=e^x$

$\left.\frac{d^2y}{dx^2}\right|_{x=\log_e^3}=e^{\log_e^3}=3$

Ans: (B)

45. If $f(1)=1,\ f'(1)=3$ then the derivative of $f\left(f\left(f(x)\right)\right)+\left(f(x)\right)^2$ at $x=1$ is

(A) 12 (B) 10 (C) 33 (D) 35

Sol: If $f(1)=1\ \ f'(1)=3$

$f\left(f\left(f(x)\right)\right)+\left(f(x)\right)^2$

Diff.

$f'f\left(f(x)\right)\cdot f'\left(f(x)\right)\cdot f'(x)+2f(x)\cdot f'(x)$

$f'f\left(f(1)\right)\cdot f'f(1)\cdot f'(1)+2f(1)\cdot f'(1)$

$f'f(1)\cdot f'(1)(3)+2(1)(3)$

$f'(1)\cdot(3)(3)+6$

$3\cdot 3\cdot 3+6=27+6=33$

Ans: (C)

46. If $y=x^{\sin x}+(\sin x)^x$ then $\frac{dy}{dx}$ at $x=\frac{\pi}{2}$ is

(A) $\frac{\pi^2}{2}$ (B) $\frac{4}{\pi}$ (C) $\pi\log\frac{\pi}{2}$ (D) 1

Sol: $y=x^{\sin x}+(\sin x)^x$

$\frac{dy}{dx}=x^{\sin x}\left(\sin x\frac{1}{x}+\log x.\cos x\right)+(\sin x)(x.\cot x+\log\sin x)$

$\frac{\pi}{2}\left(\frac{2}{\pi}+0\right)+0$

Ans: (D)

47. If $A_n=\begin{bmatrix}1-n & n\\ n & 1-n\end{bmatrix}$ then

$|A_1|+|A_2|+...+|A_{2021}=|$

(A) 4042 (B) -2021 (C) $-(2021)^2$ (D) $(2021)^2$

Sol: If $A_x=\begin{bmatrix}1-x & x\\ x & 1-x\end{bmatrix}$ then

$|A_1|+|A_2|+...+|A_{2021}|$

$\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}+\begin{vmatrix}-1 & 2\\ 2 & -1\end{vmatrix}+\begin{vmatrix}-2 & 3\\ 3 & -2\end{vmatrix}+....+\begin{vmatrix}-2020 & 2021\\ 2021 & -2020\end{vmatrix}$

$(-1)+(-3)+(-5)$

$-(1+3+5+...4041)$

$-(2021)^2$

Ans: (C)

48. If $y=\left(1+x^2\right)\tan^{-1}x-x$ then $\frac{dy}{dx}$ is

(A) $x\tan^{-1}x$ (B) $2x\tan^{-1}x$ (C) $\frac{\tan^{-1}x}{x}$ (D) $x^2\tan^{-1}x$

Sol: $y=\left(1+x^2\right)\tan^{-1}x-x$

$\frac{dy}{dx}=\left(1+x^2\right)\frac{1}{1+x^2}+\tan^{-1}x(2x)-1$

$=2x.\tan^{-1}x$

Ans: (B)

49. The co-ordinates of the point on the $\sqrt{x}+\sqrt{y}=6$ at which the tangent is equally inclined to the axes is

(A) $(6,6)$ (B) $(4,4)$ (C) $(1,1)$ (D) $(9,9)$

Sol: $\sqrt{x}+\sqrt{y}=6$

Tangent equally inclined at $x=y$

$\Rightarrow\ 2\sqrt{x}=6\ \Rightarrow\ x=9\ \Rightarrow\ y=9$

Ans: (D)

50. The function $f(x)=4\sin^3 x-6\sin^2 x+12\sin x+100$ is strictly

(A)decreasing in $\left(\frac{\pi}{2},\pi\right)$ (B)decreasing in $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

(C)decreasing in $\left[0,\frac{\pi}{2}\right]$ (D)increasing in $\left(\pi,\frac{3\pi}{2}\right)$

Sol: $f'(x)>0\ \Rightarrow 12\cos x\left(\sin^2 x-\sin x+1\right)>0$

$\Rightarrow\ \cos x>0$

(or) $f'(x)<0\ \Rightarrow\ \cos x<0\ \Rightarrow\ f$ decreases in $\left(\frac{\pi}{2},\ \pi\right)$

Ans: (A)

51. If $[x]$ is the greatest integer function not greater than x then $\int_0^8 [x]\,dx$ is equal to

(A) 20 (B) 28 (C) 30 (D) 29

Sol: $\int_0^8 [x]\,dx=\frac{8.7}{2}=28$ $\qquad \int_0^n [x]\,dx=\frac{n(n-1)}{2}$

Ans: (B)

52. $\int_0^{\pi/2}\sqrt{\sin\theta}\cos^3\theta\,d\theta$ is equal to

(A) $\frac{7}{21}$ (B) $\frac{8}{23}$ (C) $\frac{7}{23}$ (D) $\frac{8}{21}$

Sol: $\sin\theta=t$

$\int_0^1 t^{1/2}\left(1-t^2\right)dx=\left[\frac{2}{3}t^{3/2}-\frac{2}{7}t^{7/2}\right]_0^1=\frac{2}{3}-\frac{2}{7}=\frac{8}{21}$

Ans: (D)

53. If $e^y + xy = e$ the order pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at $x = 0$ is equal to

(A) $\left(\frac{-1}{e}, \frac{1}{e^2}\right)$ (B) $\left(\frac{1}{e}, \frac{1}{e^2}\right)$ (C) $\left(\frac{-1}{e}, \frac{-1}{e^2}\right)$ (D) $\left(\frac{1}{e}, \frac{-1}{e^2}\right)$

Sol: $e^y + xy = e,\ x = 0 \Rightarrow e^y = e^1 \Rightarrow y = 1$

$e^y y' + xy' + y = 0,\ x = 0,\ y = 1 \Rightarrow ey' + 1 = 0 \Rightarrow y' = \frac{-1}{e}$

$e^y y'' + e^y (y')^2 + y' + xy'' + y' = 0$

$(x = 0,\ y = 1) \Rightarrow e^{y''} + e\frac{1}{e^2} - \frac{1}{e} - \frac{1}{e} = 0$

$ey'' = \frac{1}{e} \Rightarrow y'' = \frac{1}{e^2}$

Ans: (A)

54. The function $f(x) = \log(1+x) - \frac{2x}{2+x}$ is increasing on

(A) $(-\infty, 0)$ (B) $(-\infty, \infty)$ (C) $(\infty, -1)$ (D) $(-1, \infty)$

Sol: $1 + x > 0 \Rightarrow x > -1$

$f'(x) > 0 \Rightarrow x^2 > 0 \Rightarrow x \in (-1, \infty)$

Ans: (D)

55. $\int_0^1 \frac{xe^x}{(2+x)^3} dx$ is equal to

(A) $\frac{1}{9} \cdot e - \frac{1}{4}$ (B) $\frac{1}{27} \cdot e - \frac{1}{8}$ (C) $\frac{1}{27} \cdot e + \frac{1}{8}$ (D) $\frac{1}{9} \cdot e + \frac{1}{4}$

Sol: $\int_0^1 \frac{x+2-2}{(x+2)^3} e^x\, dx = \int_0^1 \left(\frac{1}{(x+2)^2} - \frac{2}{(x+2)^3}\right) e^x\, dx = \left.\frac{e^x}{(x+2)^2}\right]_0^1 = \frac{e}{9} - \frac{1}{4}$

Ans: (A)

56. If $\int \frac{dx}{(x+2)(x^2+1)} = \mathrm{a} \log\left|1 + x^2\right| + \mathrm{b} \tan^{-1} x + \frac{1}{5} \log|x+2| + \mathrm{c}$, then

(A) $a = \frac{1}{10}\ b = \frac{-2}{5}$ (B) $a = \frac{-1}{10}\ b = \frac{2}{5}$ (C) $a = \frac{1}{10} b = \frac{2}{5}$ (D) $a = \frac{-1}{10}\ b = \frac{-2}{5}$

Sol: $\frac{1}{(x+2)(x^2+1)} = \frac{a(2x)}{1+x^2} + \frac{b}{1+x^2} + \frac{1}{5(x+2)}$

$x = 0 \Rightarrow \frac{1}{2} = b + \frac{1}{10} \Rightarrow b = \frac{1}{2} - \frac{1}{10} = \frac{2}{5}$

$x = -1 \Rightarrow \frac{1}{2} = \frac{-2a}{2} + \frac{1}{2} + \frac{1}{5}$ $\quad -a = \frac{1}{2} - \frac{2}{5} = \frac{1}{10} \Rightarrow a = \frac{-1}{10}$

Ans: (B)

57. Area of the region bounded by the curve $y = \tan x$, the $x-$axis and the line $x = \frac{\pi}{3}$ is

(A) $-\log 2$ (B) $\log\frac{1}{2}$ (C) $\log 2$ (D) 0

Sol: $\int_0^{\pi/3} \tan x \, dx = \log \sec x \Big]_0^{\pi/3} = \log 2$

Ans: (C)

58. Evaluate $\int_2^3 x^2 \, dx$ as the limit of a sum

(A) $\frac{19}{3}$ (B) $\frac{72}{6}$ (C) $\frac{53}{9}$ (D) $\frac{25}{7}$

Sol: $\int_2^3 x^2 \, dx = \left.\frac{x^3}{3}\right]_2^3 = \frac{27-8}{3} = \frac{19}{3}$

Ans: (A)

59. $\int_0^{\pi/2} \frac{\cos x \sin x}{1+\sin x} dx$ is equal to

(A) $1-\log 2$ (B) $\log 2 - 1$ (C) $\log 2$ (D) $-\log 2$

Sol: $\int_0^1 \frac{t}{1+t} \, dt = \int_0^1 \left(1 - \frac{1}{1+t}\right) dt = \left[t - \log(1+t)\right]_0^1 = 1 - \log 2$

Ans: (A)

60. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$ is equal to

(A) $2(\sin x + 2x\cos\alpha) + c$ (B) $2(\sin x - x\cos\alpha) + c$ (C) $2(\sin x + x\cos\alpha) + c$ (D) $2(\sin x - 2x\cos ga) + c$

Sol: $\int \frac{(2\cos^2 x - 1) - (2\cos^2 \alpha - 1)}{\cos x - \cos\alpha} \, dx = \int \frac{2(\cos x - \cos\alpha)(\cos x + \cos\alpha)}{\cos x - \cos\alpha} \, dx$

$= 2\int (\cos x + \cos\alpha) \, dx = 2(\sin x + x\cos\alpha) + c$

Ans: (C)

<u>Mathematics Key Answers</u>:

1. B	2. D	3. C	4. A	5.	6. D	7. C	8.	9. A	10. B
11. B	12. D	13. B	14. B	15. C	16. B	17. A	18. A	19. D	20. A
21. C	22. D	23. B	24. C	25. C	26. B	27. C	28. D	29.	30. B
31. D	32. B	33. B	34. A	35. B	36. A	37. C	38. D	39. B	40. C
41. A	42. C	43. B	44. B	45. C	46. D	47. C	48. B	49. D	50. A
51. B	52. D	53. A	54. D	55. A	56. B	57. C	58. A	59. A	60. C