KCET Board Exam – 2021

Subject: Mathematics **CODE: ________**

1. In a certain town 65% families own cell phones, 15000 families own scooter and 15% families own both. Taking into consideration that the families own at least one of the two, the total number of families in the town is

(a) 20000 (b) 30000

(c) 40000 (d) 50000

Sol: Let the total number of families in the town be x_0

Let A be the set of families who own cell phones and B be the set of families who own scooter

Then $n(A)=\frac{65x}{100}$, $n(B)=15000$, $n(A\cap B)=\frac{15x}{100}$

Now $n(A\cup B)=n(A)+n(B)-n(A\cap B)$

$x=\frac{65x}{100}+15000-\frac{15x}{100}$

$\Rightarrow x=30000$

Therefore, total number of families in the town is 30000

Ans: (b)

2. A and B are non-singleton sets and $n(A\times B)=35$. If $B\subset A$ then ${}^{n(A)}C_{n(B)}=$

(a) 28 (b) 35 (c) 42 (d) 21

Sol: $n(A\times B)=35 \Rightarrow n(A)=7$ and $n(B)=5$

$\therefore$ Required is ${}^7C_5={}^7C_2=\frac{7\times 6}{2\times 1}=21$

Ans: (d)

3. Domain of $f(x)=\frac{x}{1-|x|}$ is

(a) $R-[-1,1]$ (b) $(-\infty,1)$ (c) $(-\infty,1)\cup(0,1)$ (d) $R-\{-1,1\}$

Sol: $1-|x|\neq 0 \Rightarrow |x|\neq 1$

$\Rightarrow x\neq 1$ or -1

$\therefore$ Domain $=R-\{-1,1\}$

Ans: (d)

4. The value of $\cos 1200°+\tan 1485°$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) $-\frac{3}{2}$ (d) $-\frac{1}{2}$

Sol: GE: $\cos 1200°+\tan 1485°$

$=\cos(1080°+120°)+\tan(1440+45°)=\cos 120°+\tan 45°=-\frac{1}{2}+1=\frac{1}{2}$

Ans: (a)

5. The value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots \tan 89^\circ$ is

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) -1

Sol: GE: $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots \tan 89^\circ$

$= (\tan 1^\circ \tan 89^\circ)(\tan 2^\circ \tan 88^\circ)\ldots \tan 45^\circ$

$= 1 \times 1 \times \ldots \times 1 = 1 \qquad [\because \tan\theta \tan(90^\circ - \theta) = 1]$

Ans: (b)

6. If $\left(\frac{1+i}{1-i}\right)^x = 1$ then

(a) $x = 4n+1; n \in N$ (b) $x = 2n+1; n \in N$ (c) $x = 2n; n \in N$ (d) $x = 4n; n \in N$

Sol: $\left(\frac{1+i}{1-i}\right)^x = 1 \Rightarrow \left[\frac{(1+i)^2}{2}\right]^x = 1 \Rightarrow \left(\frac{2i}{2}\right)^x = 1$

$\Rightarrow i^x = 1 \Rightarrow x = 4n;\ n \in N$

Ans: (d)

7. The cost and revenue functions of a product are given by $C(x) = 20x + 4000$ and $R(x) = 60x + 2000$ respectively where x is the number of items produced and sold. The value of x to earn profit is

(a) >50 (b) >60 (c) >80 (d) >40

Sol: Condition $R(x) > C(x) \Rightarrow 60x + 2000 > 20x + 4000$

$\Rightarrow 40x > 2000 \Rightarrow x > 50$

Ans: (a)

8. A student has to answer 10 questions, choosing at least 4 from each of the parts A and B. If there are 6 questions in part A and 7 in part B, then the number of ways can the student choose 10 questions is

(a) 256 (b) 352 (c) 266 (d) 426

Sol: Required number $= {}^6C_4 \times {}^7C_6 + {}^6C_5 \times {}^7C_5 + {}^6C_6 \times {}^7C_4$

$= 105 + 126 + 35 = 266$

Ans: (c)

9. If the middle term of the A.P is 300 then the sum of its first 51 terms is

(a) 15300 (b) 14800 (c) 16500 (d) 14300

Sol: $T_{26} = 300 = a + 25d$

$S_{51} = \frac{51}{2}[2a + 50d] = 51(a + 25d) = 51 \times 300 = 15300$

Ans: (a)

10. The equation of straight line which passes through the point $\left(a\cos^3\theta,\ a\sin^3\theta\right)$ and perpendicular to $x\sec\theta + y\operatorname{cosec}\theta = a$ is

(a) $\frac{x}{a}+\frac{y}{a}=a\cos\theta$ (b) $x\cos\theta - y\sin\theta = a\cos 2\theta$

(c) $x\cos\theta + y\sin\theta = a\cos 2\theta$ (d) $x\cos\theta - y\sin\theta = -a\cos 2\theta$

Sol: Equation of a straight line passing through $(x_1,\ y_1)$ and perpendicular to $ax+by+c=0$ is

$b(x-x_1)-a(y-y_1)=0$

Hence the required equation is

$\operatorname{cosec}\theta\left(x-a\cos^3\theta\right)-\sec\theta\left(y-a\sin^3\theta\right)=0$

$\Rightarrow\ x\cos\theta - y\sin\theta = a\cos 2\theta$

Ans: (b)

11. The mid points of the sides of a triangle are $(1,5,-1)(0,4,-2)$ and $(2,3,4)$ then centroid of the triangle

(a) $(1,4,3)$ (b) $\left(1,4,\frac{1}{3}\right)$ (c) $(-1,4,3)$ (d) $\left(\frac{1}{3},2,4\right)$

Sol: Given, $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)=(1,5,-1)$

$\left(\frac{x_2+x_3}{2}\frac{y_2+y_3}{2}\frac{z_2+z_3}{2}\right)=(0,4,-2)$

$\left(\frac{z_3+x_1}{2}\frac{y_3+z_1}{2}\frac{z_3+z_1}{2}\right)=(2,3,4)$

$\Rightarrow\ (x_1+x_2+x_3,\ y_1+y_2+y_3,\ z_1+z_2+z_3)=(3,12,1)$

$\therefore\ G=\left(1,4,\frac{1}{3}\right)$

Ans: (b)

12. Consider the following statements:

Statement 1: $\lim\limits_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}$ is 1 (where $a+b+c\neq 0$)

Statement 2: $\lim\limits_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$ is $\frac{1}{4}$

(a) Only statement 2 is true (b) Only statement 1 is true

(c) Both statements 1 and 2 are true (d) Both statements 1 and 2 are false

Sol: Statement 1: $\lim\limits_{x\to 1}\frac{ax^2+6x+c}{cx^2+bx+a}=\frac{a+b+c}{a+b+c}=1$ (True)

Statement 2: $\lim\limits_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}=\lim\limits_{x\to -2}(-1)x^{-2}=-\frac{1}{4}\neq\frac{1}{4}$ (False)

Ans: (b)

13. If a and b are fixed non-zero constants, then the derivative of $\frac{a}{x^4}-\frac{b}{x^2}+\cos x$ is $ma+nb-p$ where

(a) $m=4x^3; n=\frac{-2}{x^3}; p=\sin x$

(b) $m=\frac{-4}{x^5}; n=\frac{2}{x^3}; p=\sin x$

(c) $m=\frac{-4}{x^5}; n=\frac{-2}{x^3}; p=-\sin x$

(d) $m=4x^3; n=\frac{2}{x^3}; p=-\sin x$

Sol: $y=\frac{a}{x^4}-\frac{b}{x^2}+\cos x$

$$y'=-\frac{4a}{x^5}+\frac{2b}{x^3}-\sin x=\left(\frac{-4}{x^5}\right)a+\left(\frac{2}{x^3}\right)b-\sin x$$

$$=ma+nb-p \Rightarrow m=\frac{-4}{x^5},\ n=\frac{2}{x^3}\quad p=\sin x$$

Ans: (b)

14. The standard deviation of the numbers $31, 32, 33 \ldots\ldots 46, 47$ is

(a) $\sqrt{\frac{17}{12}}$

(b) $\sqrt{\frac{47^2-1}{12}}$

(c) $2\sqrt{6}$

(d) $4\sqrt{3}$

Sol: $\text{SD}=\sqrt{\frac{n^2-1}{12}}=\sqrt{\frac{17^2-1}{12}}$ (where $n=17$)

$=2\sqrt{6}$

Ans: (c)

15. If $P(A)=0.59, P(B)=0.30$ and $P(A\cap B)=0.21$ then $P(A'\cap B')=$

(a) 0.11 (b) 0.38 (c) 0.32 (d) 0.35

Sol: $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=0.59+0.30-0.21=0.68$

$P(A'\cap B')=1-P(A\cup B)=1-0.68=0.32$

Ans: (c)

16. $f:R\rightarrow R$ defined by $f(x)=\begin{cases} 2x; x>3 \\ x^2 : 1<x\le 3 \\ 3x; x\le 1 \end{cases}$ then $f(-2)+f(3)+f(4)$ is

(a) 14 (b) 9 (c) 5 (d) 11

Sol: $f(-2)+f(3)+f(4)=-6+9+8=11$

Ans: (d)

17. Let $A=\{x : x\in R; x \text{ is not a positive integer}\}$ Define $f:A\rightarrow R$ as $f(x)=\frac{2x}{x-1}$, then f is

(a) injective but not surjective

(b) surjective but not injective

(c) bijective

(d) neither injective nor surjective

Sol: $f(x_1)=f(x_2) \Rightarrow x_1=x_2$

$\Rightarrow f(x)$ is one-one (injective)

f is not onto (not surjective)

For example $2 \in R$

Now $f(x) = 2$

$\Rightarrow \frac{2x}{x-1} = 2$

$\Rightarrow 2x = 2x - 2$

$\Rightarrow 0 = -2$ (not possible)

Ans: (a)

18. The function $f(x) = \sqrt{3}\sin 2x - \cos 2x + 4$ is one-one in the interval

(a) $\left[\frac{-\pi}{6}, \frac{\pi}{3}\right]$ (b) $\left(\frac{\pi}{6}, \frac{-\pi}{3}\right]$ (c) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (d) $\left[\frac{-\pi}{6}, \frac{-\pi}{3}\right)$

Sol: $f(x) = 2\left[\sin 2x\frac{\sqrt{3}}{2} - \frac{1}{2}\cos 2x\right] + 4 = 2\sin\left(2x - \frac{\pi}{6}\right) + 4$

Condition $-\frac{\pi}{2} \le 2x - \frac{\pi}{6} \le \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{6} \le x \le \frac{\pi}{3}$

Ans: (a)

19. Domain of the function $f(x) = \frac{1}{\sqrt{[x]^2 - [x] - 6}}$ where $[x]$ is greatest integer $\le x$ is

(a) $(-\infty, -2) \cup [4, \infty]$ (b) $(-\infty, -2) \cup [3, \infty]$ (c) $[-\infty, -2] \cup [4, \infty]$ (d) $[-\infty, -2] \cup (3, \infty)$

Sol: Condition $[x]^2 - [x] - 6 > 0$

$\Rightarrow ([x] - 3)([x] + 2) > 0 \Rightarrow [x] > 3$ or $[x] < -2$

$\Rightarrow x \in [4, \infty)$ or $x \in (-\infty - 2)$

$x \in (-\infty, -2) \cup [4, \infty)$

All options wrong

Ans: (a)

20. $\cos\left[\cot^{-1}\left(-\sqrt{3}\right) + \frac{\pi}{6}\right] =$

(a) 0 (b) 1 (c) $\frac{1}{\sqrt{2}}$ (d) -1

Sol: $\cos\left[\cot^{-1}\left(-\sqrt{3}\right) + \frac{\pi}{6}\right] = \cos\left[\pi - \cot^{-1}\sqrt{3} + \frac{\pi}{6}\right]$

$= \cos\left[\pi - \frac{\pi}{6} + \frac{\pi}{6}\right] = \cos\pi = -1$

Ans: (d)

21. $\tan^{-1}\left[\frac{1}{\sqrt{3}}\sin\frac{5\pi}{2}\right]\sin^{-1}\left[\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right]=$

(a) 0 (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) π

Sol: $\tan^{-1}\left[\frac{1}{\sqrt{3}}\sin\frac{5\pi}{2}\right]\sin^{-1}\left[\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right]$

$=\tan^{-1}\left[\frac{1}{\sqrt{3}}\sin\left(2\pi+\frac{\pi}{2}\right)\right]\sin^{-1}\left[\cos\left(\frac{\pi}{3}\right)\right]$

$=\tan^{-1}\left(\frac{1}{\sqrt{3}}\times 1\right)\times\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}\times\frac{\pi}{6}=\frac{\pi^2}{36}$

All options are wrong

Ans: ()

22. If $A=\begin{bmatrix}1 & -2 & 1\\ 2 & 1 & 3\end{bmatrix} B=\begin{bmatrix}2 & 1\\ 3 & 2\\ 1 & 1\end{bmatrix}$ then $(AB)'$ is equal to

(a) $\begin{bmatrix}-3 & -2\\ 10 & 7\end{bmatrix}$ (b) $\begin{bmatrix}-3 & 10\\ -2 & 7\end{bmatrix}$ (c) $\begin{bmatrix}-3 & 7\\ 10 & 2\end{bmatrix}$ (d) $\begin{bmatrix}-3 & 7\\ 10 & -2\end{bmatrix}$

Sol: $\begin{bmatrix}1 & -2 & 1\\ 2 & 1 & 3\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 2\\ 1 & 1\end{bmatrix}=\begin{bmatrix}2-6+1 & 1-4+1\\ 4+3+3 & 2+2+3\end{bmatrix}=\begin{bmatrix}-3 & -2\\ 10 & 7\end{bmatrix}\ \therefore (AB)'=\begin{bmatrix}-3 & 10\\ -2 & 7\end{bmatrix}$

Ans: (b)

23. Let M be 2×2 symmetric matrix with integer entries, then M is invertible if

(a) the first column of M is the transpose of second row of M

(b) the second row of M is the transpose of first column of M

(c) M is a diagonal matrix with non-zero entries in the principal diagonalare non-single

(d) the product of entries in the principal diagonal of M is the product of entries in the other diagonal

Sol: Take $M=\begin{bmatrix}2 & 3\\ 3 & 4\end{bmatrix}$ $2, 3, 4\in Z$

M is symmetric and inversible $[8-9=-1\neq 0]$

(a) wrong $C_1\to 2, 3\neq$ Transpose of $3, 4$

(b) wrong $R_2\to 3, 4\neq$ Transpose of $2, 3$

(c) M is diagonal matrix (wrong)

(d) Product of principle diagonal is product of the other diagonal (wrong)

All options are wrong

Ans: (c)

24. If A and B are matrices of order 3 and $|A|=5, |B|=3$ then $|3AB|$ is

(a) 425 (b) 405 (c) 565 (d) 585

Sol: $|3AB| = |3A||B| = 3|A||B| = 27\times 5\times 3 = 405$

Ans: (b)

25. If A and B are invertible matrices then which of the following is not correct?

(a) $adjA = |A|A^{-1}$ (b) $\det\left(A^{-1}\right) = \left[\det(A)\right]^{-1}$

(c) $(AB)^{-1} = B^{-1}A^{-1}$ (d) $(A+B)^{-1} = B^{-1} + A^{-1}$

Sol: Clearly (d) is wrong

Ans: (d)

26. If $f(x) = \begin{vmatrix} \cos x & 1 & 0 \\ 0 & 2\cos x & 3 \\ 0 & 1 & 2\cos x \end{vmatrix}$ then $\lim_{x\to\pi} f(x) =$

(a) -1 (b) 1 (c) 0 (d) 3

Sol: $f(x) = \cos x\left[4\cos^2 x - 3\right] = 4\cos^3 x - 3\cos x = \cos 3x$

$\therefore \lim_{x\to\pi} \cos 3x = \cos 3\pi = -1$

Ans: (a)

27. If $x^3 - 2x^2 - 9x + 18 = 0$ and $A = \begin{vmatrix} 1 & 2 & 3 \\ 4 & x & 6 \\ 7 & 8 & 9 \end{vmatrix}$ then the maximum value of A is

(a) 96 (b) 36 (c) 24 (d) 120

Sol: $A = 1(9x - 48) - 2(36 - 42) + 3(32 - 7x)$

$= 9x - 48 + 12 + 96 - 21x = -12x + 60$

$x^3 - 2x^2 - 9x + 18 = 0 \Rightarrow (x-2)(x-3)(x+3) = 0$

$\Rightarrow x = 2, 3, -3$

$\therefore$ max value of $A = -12(-3) + 60 = 96$

Ans: (a)

28. At $x = 1$, the function $f(x) = \begin{cases} x^{-3} - 1 & 1 < x < \infty \\ x - 1 & -\infty < x \le 1 \end{cases}$ is

(a) continuous and differentiable (b) continuous and non-differentiable

(c) discontinuous and differentiable (d) discontinuous and non-differentiable

Sol: $f(x) = \begin{cases} x^3 - 1 & 1 < x < \infty \\ x - 1 & -\infty < x \le 1 \end{cases}$ $f'(x) = \begin{cases} 3x^2 & 1 < x < \infty \\ 1 & -\infty < x \le 1 \end{cases}$

$\lim_{x\to 1^-} f(x) = 1 - 1 = 0$, $\lim_{x\to 1^+} f(x) = 1^3 - 1 = 0$, $f(1) = 1 - 1 = 0$

$\Rightarrow f$ is continuous at $x = 1$

$f'\left(1^-\right) = 1$ $f'\left(1^+\right) = 3$

$\Rightarrow f$ is not differentiable at $x = 1$

Ans: (b)

29. If $y=\left(\cos x^2\right)^2$, then $\frac{dy}{dx}$ is equal to

(a) $-4x\sin 2x^2$ (b) $-x\sin x^2$ (c) $-2x\sin 2x^2$ (d) $-x\cos 2x^2$

Sol: $\frac{dy}{dx}=2\cos x^2\left(-\sin x^2\right)(2x)=-2x\sin 2x^2$

Ans: (c)

30. For constant $a, \frac{d}{dx}\left(x^x+x^a+a^x+a^n\right)$ is

(a) $x^x(1+\log x)+ax^{a-1}$

(b) $x^x(1+\log x)+ax^{a-1}+a^x\log a$

(c) $x^x(1+\log x)+a^a(1+\log x)$

(d) $x^x(1+\log x)+a^a(1+\log a)+ax^{a-1}$

Sol: $\frac{d}{dx}\left[x^x+x^a+a^x+a^a\right]$

$=x^x(1+\log x)+ax^{a-1}+a^x\log a$

Ans: (b)

31. Consider the following statements:

Statement 1: If $y=\log_{10}x+\log_e x$ then $\frac{dy}{dx}=\frac{\log_{10}e}{x}+\frac{1}{x}$

Statement 2: $\frac{d}{dx}(\log_{10}x)=\frac{\log x}{\log 10}$ and $\frac{d}{dx}(\log_e x)=\frac{\log x}{\log e}$

(a) Statement 1 is true; statement 2 is false

(b) Statement 1 is false; statement 2 is true

(c) Both statements 1 and 2 are true

(d) Both statements 1 and 2 are false

Sol: Statement 1: $y=\log_{10}x+\log_e x$

$\frac{dy}{dx}=\frac{d}{dx}\left[\frac{\log_e x}{\log_e 10}\right]+\frac{1}{x}$

$=\frac{\log_{10}^{e}}{x}+\frac{1}{x}$ (True)

Statement 2: $\frac{d}{dx}\left[\log_{10}^{x}\right]=\frac{\log x}{\log 10}$ and $\frac{d}{dx}\left(\log_e^{x}\right)=\frac{\log x}{\log e}$ (wrong)

Ans: (a)

32. If the parametric equation of a curve is given by $x=\cos\theta+\log\tan\frac{\theta}{2}$ and $y=\sin\theta$, then the points for which $\frac{dy}{dx}=0$ are given by

(a) $\theta=\frac{n\pi}{2}, n\in Z$ (b) $\theta=(2n+1)\frac{\pi}{2}, n\in Z$ (c) $\theta=(2n+1)\pi, n\in Z$ (d) $\theta=n\pi, n\in Z$

Sol: $\frac{dx}{d\theta}=-\sin\theta+\frac{1}{\tan\frac{\theta}{2}}\times\sec^2\frac{\theta}{2}\times\frac{1}{2}$

$=-\sin\theta+\frac{1}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}=\frac{1}{\sin\theta}-\sin\theta=\frac{\cos^2\theta}{\sin\theta}$

$$\frac{dy}{d\theta}=\cos\theta$$

$$\therefore \frac{dy}{dx}=\frac{\cos\theta}{\frac{\cos^2\theta}{\sin\theta}}=\tan\theta=0 \Rightarrow \theta=n\pi, n\in Z$$

Ans: (d)

33. If $y=(x-1)^2(x-2)^3(x-3)^5$ then $\frac{dy}{dx}$ at $x=4$ is equal to

(a) 108 (b) 54 (c) 36 (d) 516

Sol: $\frac{dy}{dx}=2(x-1)(x-2)^3(x-3)^5+3(x-2)^2(x-1)^2(x-3)^5+5(x-3)^4(x-1)^2(x-2)^3$

$$\left(\frac{dy}{dx}\right)=48+108+360=516$$

$x=4$

Ans: (d)

34. A particle starts from rest and its angular displacement (in radians) is given by $\theta=\frac{t^2}{20}+\frac{t}{5}$. If the angular velocity at the end of $t=4$ is k, then the value of $5k$ is

(a) 0.6 (b) 5 (c) $5k$ (d) 3

Sol: $\theta=\frac{t^2}{20}+\frac{t}{5}$ $\quad \frac{d\theta}{dt}=\frac{2t}{20}+\frac{1}{5}$

$$\left.\frac{d\theta}{dt}\right|_{t=4}=k=\frac{8}{20}+\frac{1}{5}=\frac{12}{20} \Rightarrow 5k=3$$

Ans: (d)

35. If the parabola $y=\alpha x^2-6x+\beta$ passes through the point $(0,2)$ and has its tangent at $x=\frac{3}{2}$ parallel to x axis, then

(a) $\alpha=2, \beta=-2$ (b) $\alpha=-2, \beta=2$ (c) $\alpha=2, \beta=2$ (d) $\alpha=-2, \beta=2$

Sol: $y=\alpha x^2-6x+\beta \quad A=(0\ 2)$

$\therefore 2=0-0+\beta \Rightarrow \beta=2$

$y'=2\alpha x-6$

$$y'\left(\frac{3}{2}\right)=0=2\alpha\left(\frac{3}{2}\right)-6 \Rightarrow \alpha=2$$

Ans: (c)

36. The function $f(x)=x^2-2x$ is strictly decreasing in the interval

(a) $(-\infty,1)$ (b) $(1,\infty)$ (c) R (d) $(-\infty,\infty)$

Sol: $f'(x)=2x-2=2(x-1)$ $f'(x)>0 \Rightarrow x-1>0$

$\Rightarrow x>1$

$\Rightarrow x\in(1,\infty)$

Ans: (b)

37. The maximum slope of the curve $y=-x^3+3x^2+2x-27$ is

(a) 1 (b) 23 (c) 5 (d) –23

Sol: $y=-x^3+3x^2+2x-27$

$g=f'=3x^2+6x+2$

$g'=-6x+6 \qquad g''=-6<0$

$g'=0 \Rightarrow x=1$

$\therefore\ g_{\max}=-3(1)^2+6(1)+2=-3+6+2=5$

Ans: (c)

38. $\int \frac{x^3\sin\left(\tan^{-1}\left(x^4\right)\right)}{1+x^8}dx$ is equal to

(a) $\frac{-\cos\left(\tan^{-1}\left(x^4\right)\right)}{4}+C$ (b) $\frac{\cos\left(\tan^{-1}\left(x^4\right)\right)}{4}+C$

(c) $\frac{-\cos\left(\tan^{-1}\left(x^3\right)\right)}{3}+C$ (d) $\frac{\sin\left(\tan^{-1}\left(x^4\right)\right)}{4}+C$

Sol: $I=\int \frac{x^3\sin\left(\tan^{-1}x^4\right)}{1+x^8}dx$

$t=\tan^{-1}x^4 \Rightarrow \frac{dt}{4}=\frac{x^3dx}{1+x^8}$

$I=\frac{1}{4}\int \sin t\,dt=-\frac{1}{4}\cos t+C \quad =-\frac{1}{4}\cos\left[\tan^{-1}x^4\right]+C$

Ans: (a)

39. The value of $\int \frac{x^2dx}{\sqrt{x^6+a^6}}$ is equal to

(a) $\log\left|x^3+\sqrt{x^5+a^5}\right|+C$ (b) $\log\left|x^3-\sqrt{x^6+a^6}\right|+C$

(c) $\frac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right|+C$ (d) $\frac{1}{3}\log\left|x^3-\sqrt{x^6+a^6}\right|+C$

Sol: $I=\int \frac{x^2dx}{\sqrt{x^6+a^6}}=\frac{1}{3}\int \frac{dt}{\sqrt{t^2+a^6}}$ (put $x^3=t$)

$\Rightarrow x^2dx=\frac{dt}{3}$

$=\frac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right|+C$

Ans: (c)

40. The value of $\int \frac{xe^xdx}{(1+x)^2}$ is equal to

(a) $e^x(1+x)+C$ (b) $e^x\left(1+x^2\right)+C$ (c) $e^x(1+x)^2+C$ (d) $\frac{e^x}{1+x}+C$

Sol: $\int \frac{x}{(1+x)^2} e^x dx = \int \left[\frac{1}{1+x} - \frac{1}{(1+x)^2}\right] e^x dx \quad = \frac{e^x}{1+x} + C$

Ans: (d)

41. The value of $\int e^x \left[\frac{1+\sin x}{1+\cos x}\right] dx$ is equal to

(a) $e^x \tan\frac{x}{2} + C$ (b) $e^x \tan x + C$ (c) $e^x (1+\cos x) + C$ (d) $e^x (1+\sin x) + C$

Sol: $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx = \int e^x \left[\tan\frac{x}{2} + \frac{1}{2}\sec^2\frac{x}{2}\right] dx \quad = e^x \tan\frac{x}{2} + C$

Ans: (a)

42. If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$ where n is positive integer then $I_{10} + I_8$ is equal to

(a) 9 (b) $\frac{1}{7}$

(c) $\frac{1}{8}$ (d) $\frac{1}{9}$

Sol: $I_n = \int_0^{\pi/4} \tan^n x \, dx$

$I_{10} + I_8 = \int_0^{\pi/4} \tan^{10} x \, dx + \int_0^{\pi/4} \tan^8 x \, dx$

$= \int_0^{\pi/4} \tan^8 x \left(\tan^2 x + 1\right) dx$

$= \int_0^{\pi/4} \tan^8 x \sec^2 x \, dx$

$= \frac{t^9}{9} = \left.\frac{\tan^9 x}{9}\right]_0^{\pi/4}$ put $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$= \frac{1}{9}$

Ans: (d)

43. The value of $\int_0^{4042} \frac{\sqrt{x}\, dx}{\sqrt{x} + \sqrt{4042-x}}$ is equal to

(a) 4042 (b) 2021

(c) 8084 (d) 1010

Sol: Let $I = \int_0^{4042} \frac{\sqrt{x}\, dx}{\sqrt{x} + \sqrt{4042-x}}$ … (1)

Then $I = \int_0^{4042} \frac{\sqrt{4042-x}}{\sqrt{4042-x} + \sqrt{x}} dx$ … (2)

Adding (1) and (2)

$$2I = \int_0^{4042} dx = x\Big]_0^{4042}$$

$2I = 4042$

$\Rightarrow \ I = \dfrac{4042}{2} = 2021$

Ans: (b)

44. The area of the region bounded by $y = \sqrt{16 - x^2}$ and x axis is

(a) 8π square units (b) 20π square units

(c) 16π square units (d) 256π square units

Sol: $y = \sqrt{16 - x^2}$ and x- axis

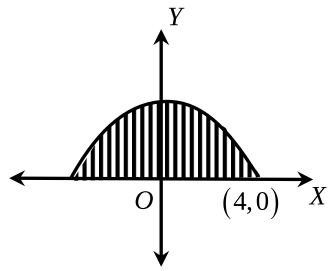
$x^2 + y^2 = 16 = 4^2$

$\Rightarrow$ Required area is area of circle

$= \pi(4)^2 = 16\pi$

Since y is positive

So, area $= 8\pi$ sq. units



Ans: (a)

45. If the area of the Ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{\lambda^2} = 1$ is 20π square units, then λ is

(a) ± 4 (b) ± 3 (c) ± 2 (d) ± 1

Sol: $\dfrac{x^2}{25} + \dfrac{y^2}{\lambda^2} = 1 \quad \Rightarrow a = 5,\ b = \lambda$

$A = \pi ab \Rightarrow \pi(5)(\lambda) = 20 \Rightarrow \lambda = 4$

Ans: (a)

46. Solution of the differential equation $xdy - ydx = 0$ represents

(a) A rectangular hyperbola (b) Parabola whose vertex is at origin

(c) Straight line passing through origin (d) A circle whose centre is origin

Sol: $xdy - ydx = 0$

$\Rightarrow \dfrac{dy}{y} = \dfrac{dx}{x}$

Integrating on both sides

$\log y = \log x + \log c$

$\Rightarrow y = xc$

$\Rightarrow$ Straight line passing through the origin

Ans: (c)

47. The number of solutions of $\dfrac{dy}{dx} = \dfrac{y+1}{x-1}$ when $y(1) = 2$ is

(a) three (b) one (c) infinite (d) two

Sol: $\frac{dy}{y+1}=\frac{dx}{x-1}$, $x=1$, $y=2$

$\log(y+1)=\log(x-1)+\log C=\log[C(x-1)]$

$y+1=C(x-1)$

When $x=1$, $y=2$

$\Rightarrow 3=C(0)\Rightarrow y+1=3(x-1)$

Equation has infinite solution.

Ans: (c)

48. A vector $\vec{a}$ makes equal acute angles on the coordinate axis. Then the projection of vector $\vec{b}=5\hat{i}+7\hat{j}-\hat{k}$ on $\vec{a}$ is

(a) $\frac{11}{15}$ (b) $\frac{11}{\sqrt{3}}$ (c) $\frac{4}{5}$ (d) $\frac{3}{5\sqrt{3}}$

Sol: $\vec{a}=\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}$

Required $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=\frac{\frac{1}{\sqrt{3}}(5)+7\left(\frac{1}{\sqrt{3}}\right)-\frac{1}{\sqrt{3}}}{\sqrt{75}}$

$=\frac{11}{\sqrt{3}\cdot 5\sqrt{3}}=\frac{11}{15}$

Ans: (a)

49. The diagonals of a parallelogram are the vectors $3\hat{i}+6\hat{j}-2\hat{k}$ and $-\hat{i}-2\hat{j}-8\hat{k}$ then the length of the shorter side of parallelogram is

(a) $2\sqrt{3}$ (b) $\sqrt{14}$ (c) $3\sqrt{5}$ (d) $4\sqrt{3}$

Sol: $\vec{a}+\vec{b}=3\hat{i}+6\hat{j}-2\hat{k}$

$\vec{a}-\vec{b}=-\hat{i}-2\hat{j}-3\hat{k}$

$2\vec{a}=2\hat{i}+4\hat{j}-10\hat{k}\Rightarrow \vec{a}=\hat{i}+2\hat{j}-5\hat{k}$

$2\vec{b}=4\hat{i}+8\hat{j}+6\hat{k}\Rightarrow \vec{b}=2\hat{i}+4\hat{j}+3\hat{k}$

$|\vec{a}|=\sqrt{1+4+25}=\sqrt{30}$

$|\vec{b}|=\sqrt{4+16+9}=\sqrt{29}$

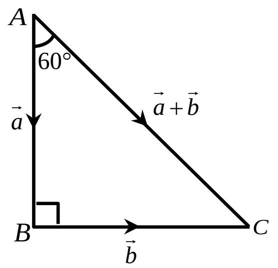
All options are wrong

Ans: ()

50. If $\vec{a}\cdot\vec{b}=0$ and $\vec{a}+\vec{b}$ makes an angle $60°$ with $\vec{a}$ then

(a) $|\vec{a}|=2|\vec{b}|$ (b) $2|\vec{a}|=|\vec{b}|$ (c) $|\vec{a}|=\sqrt{3}|\vec{b}|$ (d) $\sqrt{3}|\vec{a}|=|\vec{b}|$

Sol:



$\tan 60° = \frac{BC}{AB} \Rightarrow BC = \sqrt{3}AB$

$|\vec{b}| = \sqrt{3}|\vec{a}|$

Ans: (d)

51. If the area of the parallelogram with $\vec{a}$ and $\vec{b}$ as two adjacent sides is $15\,\text{sq}$. units then the area of the parallelogram having $3\vec{a}+2\vec{b}$ and $\vec{a}+3\vec{b}$ as two adjacent sides in sq. units is

(a) 45 (b) 75 (c) 105 (d) 120

Sol: $|\vec{a}\times\vec{b}| = 15$

$(3\vec{a}+2\vec{b})\times(\vec{a}+3\vec{b})$

$= 9\vec{a}\times\vec{b} + 2\vec{b}\times\vec{a} = 7\vec{a}\times\vec{b}$

$\therefore$ Required area

$= 7|\vec{a}\times\vec{b}| = 105$

Ans: (c)

52. The equation of the line joining the points $(-3,4,11)$ and $(1,-2,7)$ is

(a) $\frac{x+3}{2} = \frac{y-4}{3} = \frac{z-11}{4}$ (b) $\frac{x+3}{-2} = \frac{y-4}{3} = \frac{z-11}{2}$

(c) $\frac{x+3}{-2} = \frac{y+4}{3} = \frac{z+11}{4}$ (d) $\frac{x+3}{2} = \frac{y+4}{-3} = \frac{z+11}{2}$

Sol: Required equation of line

$\frac{x+3}{1+3} = \frac{y-4}{-2-4} = \frac{2-11}{7-11}$

Or $\frac{x+3}{4} = \frac{y-4}{-6} = \frac{2-11}{-4}$ Or $\frac{x+3}{-2} = \frac{y-4}{3} = \frac{z-11}{2}$

Ans: (b)

53. The angle between the lines whose direction cosines are $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{-\sqrt{3}}{2}\right)$ is

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

Sol: $\cos\theta = \left(\frac{\sqrt{3}}{4}\right)\left(\frac{\sqrt{3}}{4}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{4}\right) - \frac{3}{4} = \frac{3}{16} + \frac{1}{16} - \frac{12}{16} = \frac{-8}{16} = -\frac{1}{2}$

$\therefore$ Acute angle $\cos\theta=\frac{1}{2}\Rightarrow\ \theta=\frac{\pi}{3}$

Ans: (c)

54. If a plane meets the coordinate axes at A, B and C in such a way that the centroid of triangle ABC is at the point $(1, 2, 3)$ then the equation of the plane is

(a) $\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=1$ (b) $\frac{x}{3}+\frac{y}{6}+\frac{z}{9}=1$ (c) $\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=\frac{1}{3}$ (d) $\frac{x}{1}-\frac{y}{2}+\frac{z}{3}=-1$

Sol: $A=(a,0,0)\ B=(0,b,0)\ C=(0,0,c)$

$G=\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)=(1,2,3)\Rightarrow a=3,\ b=6,\ c=9$

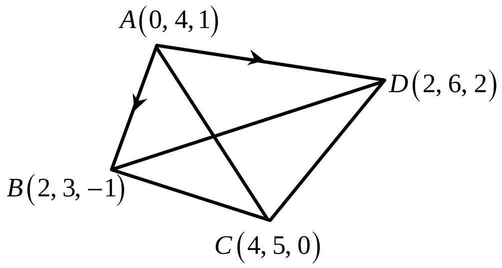
$\therefore$ Equation is $\frac{x}{3}+\frac{y}{6}+\frac{z}{9}=1$

Ans: (b)

55. The area of the quadrilateral $ABCD$, when $A(0,4,1)B(2,3,-1)C(4,5,0)$ and $D(2,6,2)$ is equal to

(a) 9 sq. units (b) 18 sq. units (c) 27 sq. units (d) 81 sq. units

Sol: Midpoint of AC = Midpoint of BD



$\Rightarrow$ $ABCD$ is a parallelogram

$\overrightarrow{AB}=(2-0)\hat{i}+(3-4)\hat{j}+(-1-1)\hat{k}=2\hat{i}-\hat{4}-2\hat{k}$

$\overrightarrow{AD}=(2-0)\hat{i}+(6-4)\hat{j}+(2-1)\hat{k}=2\hat{i}+2\hat{j}+\hat{k}$

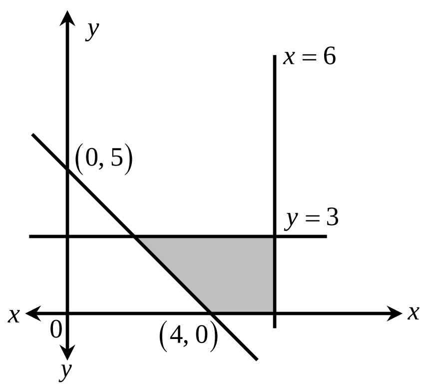
$$\therefore\ \overrightarrow{AB}\times\overrightarrow{AD}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & -2\\ 2 & 2 & 1\end{vmatrix}$$

$=\hat{i}(-1+4)-\hat{j}(2+4)+\hat{k}(4+2)=3\hat{i}-6\hat{j}+6\hat{k}$

$\therefore$ Area $=\left|3\hat{i}-6\hat{j}+6\hat{k}\right|=81$

Ans: (d)

56. The shaded region is the solution set of the inequalities



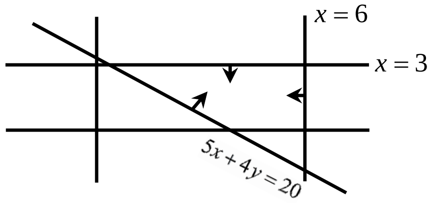
(a) $5x+4y\geq 20,\ x\leq 6,\ y\geq 3,\ x\geq 0,\ y\geq 0$

(b) $5x+4y\leq 20,\ x\leq 6,\ y\leq 3,\ x\geq 0,\ y\geq 0$

(c) $5x+4y\geq 20,\ x\leq 6,\ y\leq 3,\ x\geq 0,\ y\geq 0$

(d) $5x+4y\geq 20,\ x\geq 6,\ y\leq 3,\ x\geq 0,\ y\geq 0$

Sol:



Required region $5x + 4y \geq 20$

$x \leq 6,\ \ y \leq 3,\ \ x \geq 0,\ \ y \geq 0$

Ans: (c)

57. Given that A and B are two events such that $P(B)=\frac{3}{5}P(A/B)=\frac{1}{2}$ and $P(A\cup B)=\frac{4}{5}$ then $P(A)=$

(a) $\frac{3}{10}$ (b) $\frac{1}{2}$ (c) $\frac{1}{5}$ (d) $\frac{3}{5}$

Sol: $P(B)=\frac{3}{5}P(A/B)=\frac{1}{2}$

$P(A\cup B)=\frac{4}{5}$

$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)+P(B)-P(A\cup B)}{P(B)}$

$\therefore\ \frac{1}{2}=\frac{P(A)+\frac{3}{5}-\frac{4}{5}}{\frac{3}{5}} \Rightarrow P(A)=\frac{5}{10}=\frac{1}{2}$

Ans: (b)

58. If A, B and C are three independent events such that $P(A)=P(B)=P(C)=P$ then P (at least two of A, B, C occur) $=$

(a) P^3-3P (b) $3P-2P^2$ (c) $3P^2-2P^3$ (d) $3P^2$

Sol: Required Prob $=P(A')P(B)P(C)+P(A)P(B')P(C)+P(A)P(B)P(C')+P(A)P(B)P(C)$

$=(1-P)P^2+(1-P)P^2+(1-P)P^2+P^3$

$=P^2-P^3+P^2-P^3+P^2-P^3+P^3$

$=3P^2-2P^3$

Ans: (c)

59. Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6 then the probability of getting a sum as 3 is

(a) $\frac{1}{18}$ (b) $\frac{5}{18}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

Sol: A: getting sum less than 6

$A=\{(1,4)(1,3)(1,2)(1,1),(2,3)(2,2)(2,1),(3,2)(3,1)(4,1)\}$ $n(A)=10$

B: getting sum 3

$B=\{(1,2),(2,1)\}$, $n(B)=2$

$A \cap B = \{(1,2),(2,1)\} = B$

$\therefore$ Required probability $P(B/A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{2}{10} = \dfrac{1}{5}$

Ans: (c)

60. A car manufacturing factory has two plants X and Y. Plant X manufactures 70% of cars and plant Y manufactures 30% of cars. 80% of cars at plant X and 90% of cars at plant Y are rated as standard quality. A car is chosen at random and is found to be of standard quality. The probability that its has come from plant X is

(a) $\dfrac{56}{73}$ (b) $\dfrac{56}{84}$ (c) $\dfrac{56}{83}$ (d) $\dfrac{56}{79}$

Sol: $P(X) = \dfrac{70}{100}$ $P(Y) = \dfrac{30}{100}$

$P(S/X) = \dfrac{80}{100}$ $P(S/Y) = \dfrac{90}{100}$

By Baye's theorem

$$P(X/S) = \frac{P(X)P(S/X)}{P(X)P(S/X) + P(Y)P(S/Y)} = \frac{\frac{70}{100} \times \frac{80}{100}}{\frac{70}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{90}{100}} = \frac{56}{83}$$

Ans: (c)

<u>Key Answers</u>:

1. b	2. d	3. d	4. a	5. b	6. d	7. a	8. c	9. a	10. b
11. b	12. b	13. b	14. c	15. c	16. d	17. a	18. a	19. a	20. d
21.	22. b	23. c	24. b	25. d	26. a	27. a	28. b	29. c	30. b
31. a	32. d	33. d	34. d	35. c	36. b	37. c	38. a	39. c	40. d
41. a	42. d	43. b	44. a	45. a	46. c	47. c	48. a	49.	50. d
51. c	52. b	53. c	54. b	55. d	56. c	57. b	58. c	59. c	60. c

21. All options are wrong   49. All options are wrong