

Chapter-02

Polynomials

- An algebraic expression of the form $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n$, where $a_0, a_1, a_2, \dots, a_n$ are real numbers, n is a non-negative integer and $a_0 \neq 0$ is called a polynomial of degree n .
- **Degree:** The highest power of x in a polynomial $p(x)$ is called the degree of polynomial.
- Polynomials of degrees 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.
- **Types of Polynomial:**
 - (i) **Constant Polynomial:** A polynomial of degree zero is called a constant polynomial and it is of the form $p(x) = k$.
 - (ii) **Linear Polynomial:** A polynomial of degree one is called linear polynomial and it is of the form $p(x) = ax + b$ where a, b are real numbers and $a_0 \neq 0$.
 - (iii) **Quadratic Polynomial:** A quadratic polynomial in x with real coefficient is of the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.
 - (iv) **Cubical Polynomial:** A polynomial of degree three is called cubical polynomial and is of the form $p(x) = ax^3 + bx^2 + cx + d$ where a, b, c, d are real numbers and $a \neq 0$.
 - (v) **Bi-quadratic Polynomial:** A polynomial of degree four is called bi-quadratic polynomial and it is of the form $p(x) = ax^4 + bx^3 + cx^2 + dx + e$, where a, b, c, d, e are real numbers and $a \neq 0$.
- The zeroes of a polynomial $p(x)$ are precisely the x -coordinates of the points where the graph of $y = p(x)$ intersects the x -axis i.e. $x = a$ is a zero of polynomial $p(x)$ if $p(a) = 0$.
- A polynomial can have at most the same number of zeros as the degree of polynomial.
- For quadratic polynomial $ax^2 + bx + c \neq 0$ Sum of zeros = $-\frac{b}{a}$ Product of zeros = $\frac{c}{a}$
- The division algorithm states that given any polynomial $p(x)$ and polynomial $g(x)$, there are polynomials $q(x)$ and $r(x)$ such that: $p(x) = g(x).q(x) + r(x)$, $g(x) \neq 0$ where $r(x) = 0$ or degree of $r(x) < \text{degree of } g(x)$
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